Free surface lubrication of rotating cylinders by impacting Newtonian liquid jet

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Abstract

Free surface lubrication of rotating cylinders by Newtonian liquid jets was investigated at different impingement positions, jet- and circumferential velocities. The interaction between impinging round laminar jet and rotating surface was characterized by high-speed imaging. Compared to flat surface impingement, the liquid deposition mechanism is more complex and often accompanied by phenomena such as splashing, jet deflection and formation of radial surface stripes. Liquid splashing initially develops in direct deposition regime due to hydraulic jump-induced droplet shedding, transitioning to lamella instability splash at higher Reynolds numbers. The critical Reynolds number for splash onset is of same order of magnitude as for flat surface impingement and fairly constant at low relative jet impingement angles, but increases exponentially when angle approaches 90°. Liquid film width and thickness were both determined proportional to the square root of the jet/surface velocity ratio, with respective power law models in good agreement with measured values.

Keywords: liquid jet impingement; rotating surface; free surface lubrication; splashing; hydraulic jump; lamella instability

1 Introduction

Uniform lubrication of moving and rotating surfaces by liquid jet impingement and subsequent deposition is essential in many industrial applications that involve mass or heat transfer. Most common examples include impingement cooling in the steel industry [1][2], shaft and bearing lubrication [3], application of liquid friction modifiers in railway transport [4][5], high-speed melt spinning [6] and cascading melt flow on spinning machines in production of mineral wool [7][8]. Many industrial applications of impinging liquid jets require precise control of the liquid deposition process to achieve uniform properties of lubrication liquid layer and minimize liquid loss through splashing. The

fundamental problem of liquid impaction on a free surface and resulting splashing has been addressed by many studies. For instance, Rioboo et al. [9] investigated the droplet splash against a stationary dry surface, identifying six different mechanisms of interaction between droplets and the substrate. An even more in-depth analysis of droplet splashing-related phenomena was provided by Deegan et al. [10], while Choo and Kim [11] experimented with round jet impaction on a still substrate and associated hydraulic jump.

Having said that, the mechanism of liquid deposition and splashing becomes significantly more complex when droplets or jets impact on a moving free surface, i.e. when liquid tangential velocity in the reference frame of the surface is significant. Relevant droplet impact research was performed for single droplets [12] as well as sprays [13] and droplet trains [14]. Each of these liquid structures possesses a unique surface impingement behavior, and so do continuous liquid jets that will be the focus of this paper. In the last decade, there has been a growing interest in phenomena of jet-surface impaction with nonzero tangential velocity due to novel industrial applications such as liquid friction modifiers in rail transport [15]. The mechanism of jet deposition was investigated for both Newtonian [4][16][17][18] and non-Newtonian liquids [17][19]. In Newtonian liquids, the most decisive parameters for occurrence of splash are the jet and substrate velocity, substrate surface roughness to jet diameter ratio [4], as well as ambient air pressure [17]. As explained by Keshavarz et al. [19], splashing is generated as the portion of the liquid film deposited upstream of the impingement point, also known as lamella, is lifted from the substrate due to excessive tangential jet velocity in the substrate frame of reference.

In contrast to jet impaction on a flat moving surface, few studies exist for impingement on the mantle surface of a rotating cylinder despite existing and prospective applications in areas such as melt spinning, train wheel surface lubrication with liquid friction modifiers and mineral melt fiberization on spinners. The flow of annular films on external cylinder surface has been quite extensively studied and modelled [20], albeit mostly for other methods of liquid application such as partial cylinder immersion in a liquid bath. Only recently, Bizjan et al. [21] investigated impingement splashing intensity on a twin wheel atomizer as a function of the liquid flow rate and impingement position. A decisive role of hydrodynamic phenomena such as hydraulic jump and jet deflection on splashing was identified. A study by [4] has also brought to our attention the necessity to consider the effect of impinging jet velocity on liquid deposition efficiency, and employ a dimensionless formulation of operating conditions comparable to existing studies for flat surface impingement. The present paper aims to address all of these considerations.

2 Materials and methods

Experiments were conducted on a measuring station shown in Fig. 1. The measurement setup comprised a rotating cylinder, a liquid feeding system and a high-speed imaging setup. The aluminum cylinder with $2R = 90$ mm diameter was spun about a horizontal rotation axis using a three-phase asynchronous AC motor. The rotational speed of the electric motor was varied between 0 Hz and 100 Hz by means of a variable frequency drive (VFD). To investigate liquid jet impingement and deposition phenomena, the liquid (mixture of 65% sucrose and 35% water) was supplied to a nozzle with $D_0 = 3.0$ mm inner diameter that discharged a round laminar jet 132 mm above the cylinder surface. Liquid properties at 25°C ambient temperature were as follows [22]: density 1315 kg/m³, viscosity 105 mPa s and surface tension 73.3 mN/m. The working medium was purposefully selected to have a relatively high viscosity in order to minimize the slippage of liquid against the cylinder surface [21], and to ensure laminar flow in the free-jet and impingement zone that is preferable for splash-free deposition process [23].

Fig. 1. Experimental setup for jet impact imaging

To obtain a sufficiently wide range of jet velocities for a comprehensive study, the liquid was supplied to the nozzle by a VFD-powered gear pump, and its flow rate could be controlled by manipulation of either the pump motor speed, or the ratio of liquid returning to the tank via the bypass line. The mass flow rate $q = Q/\rho$ (Fig. 2) was measured by weighing of liquid discharged in 20 s. The transversal position of jet impingement (angle $0^\circ \leq \varphi < 90^\circ$) was adjustable by a linear positioning mechanism.

As noted by Range and Feuillebois [24] and Keshavarz et al. [4], the threshold for occurrence of liquid splashing is heavily dependent on the roughness of the surface. Since liquid deposition phenomena are of interest in many technological processes, surface properties were selected to resemble typical conditions in the industry. The mantle surface of the cylinder was processed to an average surface roughness height of $\varepsilon = 35 \mu\text{m}$ and was wetted by a thin film upstream of the impingement point, as fast-spinning cylinders are typically wetted along their complete perimeter in most industrial applications. The wetting was carried out by a polyurethane foam sponge mounted at the side of the cylinder that removed excessive deposited liquid and assured that a thin film (thickness estimated to 0.2 mm) was uniformly spread across the complete 60 mm length of the cylinder. Jet impaction can be significantly affected by presence of substrate coating with a liquid film when the film is thick and of low viscosity [2]. Nevertheless, if the liquid film coating is thin and viscous (as was the case in our experiments), the slippage of liquid against the cylinder and between layers of coating and jet-deposited film can be neglected [25]. Consequently, we do not expect deposition mechanism to differ significantly from the dry surface impingement.

Fig. 2. Mass flow rate and mean nozzle exit velocity of liquid as a function of pump rotational speed

The process of liquid deposition was recorded by a high-speed camera (Fastec Hispec 4 Mono with 50 mm lenses) at 3200 frames per second and with 736x576 pixels image size (pixel size was 59.3 μm). Cylinder surface and adjacent liquid structures were illuminated from the side by an LED panel. In the first set of experiments, operating conditions were kept constant during each particular recording. Then, another set of experiments was conducted with variable cylinder surface velocity at a constant liquid jet velocity, and vice versa to more accurately determine critical velocities and corresponding dimensionless parameters above which splashing occurs.

3 Results and discussions

The wide range in which the cylinder surface and liquid jet velocity at impingement point (v_s and v_j , respectively) were varied resulted in a plentitude of different deposition regimes. Typically, a combination of low surface and jet velocity produce direct liquid deposition without splashing (Fig. 3a), while an increase in either of these velocities leads to occurrence of splash (Fig. 3b-d). If, in a direct deposition regime, jet velocity is increased (Fig. 3c), a stable liquid sheet is formed due to lamella instability (fundamental mechanism discussed by Bird et al. [26]), representing a substantial reduction in liquid deposition efficiency. When surface velocity is increased instead, the hydraulic jump forming upstream of the impingement point [1] is destabilized, resulting in droplet shedding downstream. Above a certain critical rotation speed of a cylinder, a large portion of impinging jet can be deflected (Fig. 3b) and thus not deposited on the surface, which can be harnessed to deposit liquid on multiple successive cylinders [21].

Regardless of the presence of jet deflection, droplet shedding is undesired as it reduces the efficiency of liquid deposition. The droplet shedding phenomenon is mainly caused by the rim instability that is known to occur on impact of droplets and jets on a surface [27][28]. As demonstrated by Thunivumani and Gadgil [18] in splash plate impingement studies, the impact angle between the jet and the surface can have a significant effect on liquid spreading angle and rim instability downstream of the impingement point. Therefore, our experiments were conducted at impingement positions ranging between $\varphi = 0^\circ$ and $\varphi = 60^\circ$ to investigate the effects on liquid splashing, with flow imaging performed at $\varphi = 15^\circ$ and $\varphi = 30^\circ$.

Having said that, there are also other factors contributing to the occurrence of splashing. If the rotating cylinder surface is wetted upstream of the impingement point (as was the case in our experiments), radial stripes appear on the liquid film due to the ring instability [29], leading to a situation similar to jet impact on a grooved cylinder surface. In Fig. 3c, interaction between stripes and liquid sheet can be observed as discontinuities on the otherwise stable sheet. From this point onwards, either a reduction in v_j (Fig. 3d) or an increase in v_s destabilizes the sheet, causing it to intermittently break up and reducing its extent upstream of the impingement point. Nevertheless, the height, width and spacing of liquid stripes can be seen to drop as the speed of the cylinder surface is increased (Fig. 3b), suggesting a diminishing effect on liquid deposition and splashing.

Fig. 3. Different jet impingement regimes: a) direct deposition ($v_s = 7.92$ m/s, $v_j = 1.94$ m/s, $\varphi = 30^\circ$, $Re = 215$, $We = 2.20 \cdot 10^3$); b) splashing by rim instability, jet deflection ($v_s = 19.4$ m/s, $v_j = 1.94$ m/s, $\varphi = 15^\circ$, $Re = 570$, $We = 1.55 \cdot 10^4$); c) splashing by lamella instability, stable sheet ($v_s = 7.92$ m/s, $v_j = 9.23$ m/s, $\varphi = 30^\circ$, $Re = 260$, $We = 3.22 \cdot 10^3$); d) splashing by lamella instability, unstable and broken sheet ($v_s = 7.92$ m/s, $v_j = 3.77$ m/s, $\varphi = 30^\circ$, $Re = 206$, $We = 2.03 \cdot 10^3$)

Considering experimental insights provided so far, it is evident that liquid deposition on a rotating cylinder is significantly more complex and depending on a larger number of operating condition than deposition on a flat moving surface. On a rotating cylinder, a forming liquid film is also subjected to centrifugal force, unlike jet impaction on a flat surface. To reduce the amount of significant parameters for result presentation and allow for better comparability of results with future studies, dimensionless Reynolds and Weber numbers will be introduced (Eqs. (1) and (2), respectively), along with the relative impingement angle α (Eq. (6)).

Reynolds number:

$$Re = \frac{\rho v_R D}{\mu} \quad (1)$$

Weber number:

$$We = \frac{\rho v_R^2 D}{\sigma} \quad (2)$$

In Eqs. (1) and (2), v_R is relative velocity in the moving frame of reference of cylinder surface (Fig. 4), and D is jet diameter at impingement point (measured from high-speed images). When the jet contraction ratio D/D_0 is determined, jet velocity at impact (v_j) can be calculated from the continuity equation assuming incompressible liquid and mass conservation from nozzle outlet (cross section $A_0 = \pi D_0^2/4$) to cylinder surface (cross section $A = \pi D^2/4$) – Eqs. (3) and (4):

$$Q = v_{j0} A_0 = v_j A \quad (3)$$

In our experiments, the jet contraction ratio was fairly constant, $D/D_0 = 0.818 \pm 0.038$. Assuming a circular cross-section of the jet at impingement, proportionality $A \propto D^2$ is valid and v_j can be expressed from Eq. (3) as follows:

$$v_j = v_{j0} \left(\frac{D_0}{D} \right)^2 \quad (4)$$

To investigate velocity conditions at the impingement point, the rotating cylinder surface will be approximated by a flat plate inclined by an angle φ with respect to horizontal axis and moving with velocity vector $\mathbf{v}_s$ (Fig. 4). The magnitude of cylinder surface velocity is equal to the circumferential velocity at radius R , $v_s = 2\pi f R$. Its x and y components can be calculated as $v_{sx} = v_s \cos(\varphi)$ and $v_{sy} = v_s \sin(\varphi)$, respectively (consider Fig. 4 for graphical presentation).

Fig. 4. Jet impingement in absolute (a) and relative (b) coordinate system, definition of relative impingement velocity and angle

Now, the vector of relative jet impingement velocity can be written as $\mathbf{v}_R = (-v_{sx}, v_j - v_{sy})$. Its magnitude can be expressed by Eq. (5):

$$v_R = \sqrt{v_{sx}^2 + (v_j - v_{sy})^2} = \sqrt{(2\pi f R \cos \varphi)^2 + (4q/(\pi D^2 \rho) - 2\pi f R \sin \varphi)^2} \quad (5)$$

Besides v_R , another key parameter for assessment of jet impact onto the moving surface is the impingement angle in the frame of reference of cylinder surface, $\alpha = \beta - \varphi$. Considering geometric relations presented in Fig. 4, angles α and β can be expressed by Eqs. (6) and (7), respectively:

$$\beta = \text{atan}\left(\frac{v_{sx}}{v_j - v_{sy}}\right) = \text{atan}\left(\frac{\cos\varphi}{2q/(\pi^2 D^2 f R \rho) - \sin\varphi}\right) \quad (6)$$

$$\alpha = \text{atan}\left(\frac{\cos\varphi}{2q/(\pi^2 D^2 f R \rho) - \sin\varphi}\right) - \varphi \quad (7)$$

The relative impingement angle can assume values in the range $-\varphi < \alpha < 90^\circ$. In Fig. 5, scalar fields of dimensionless parameters α and Re are calculated for v_j and v_s in expected experimental range of 0...30 m/s, and for two different jet impingement angles.

Fig. 5. Dependence of dimensionless parameters α and Re on v_j , v_s and φ for $R = 45\text{mm}$

It can be noted that for the same range of jet and surface velocity, the map of Reynolds number distribution changes with impingement angle, reaching a maximum of $Re = 1275$ for $\varphi = 0^\circ$ and $Re = 902$ for $\varphi = 30^\circ$, implying laminar flow under all experimental conditions. Reynolds number isolines in Fig. 5 are proportional to curves of constant relative velocity, which is equal to both v_j and v_s axis values in intersection points with a selected Re curve (e.g. $v_R = 3.4$ m/s when $Re = 100$, irrespective of φ). Since splashing typically occurs above a certain Re (or equally, v_R) value for a given α angle [4], Re isolines can be interpreted as operating regions equally prone to splashing (for sufficiently low relative impingement angles – say, $\alpha < 45^\circ$, as we will later demonstrate in Fig. 9). Another observation that arises from Fig. 5 and Eq. (7) is that in the case of a still cylinder, $\alpha = -\varphi$ regardless of v_j , while for $v_s \gg v_j$, α limits towards 90° . The latter situation is typical for melt film formation on mineral wool spinning machines, where melt jet is gravity-fed and thus slow ($v_j \approx 2$ m/s), while the cylinder surface has a high circumferential velocity ($v_s \approx 100$ m/s).

In Fig. 6, α is plotted with respect to the Reynolds number as defined by Eq. (1). Results presented in Fig. 6 imply a rather complex relation between Reynolds number and angle α , unlike impingement on a dry flat surface where deposition and splash region are clearly delimited (Keshavarz et al. [4], dashed line in Fig. 6). Direct deposition of liquid was observed at a combination of low or negative α values and moderate Reynolds numbers ($250 < Re < 350$), as well as for operating points with $v_j=1.94$ m/s, $\varphi = 30^\circ$, $Re < 300$ and $\alpha > 40^\circ$. Under other operating conditions presented in Fig. 6, liquid jet splashed on contact with the moving surface.

Fig. 6. Relative jet impingement angle and Weber number as a function of Reynolds number

The first possible splashing mechanism is droplet shedding due to the rim instability downstream of the hydraulic jump zone (Fig. 3b). Droplet shedding was particularly intense when a combination of high Re and α was used. Such conditions also resulted in jet deflection from the cylinder surface (Fig. 4b), with only a low fraction of the total flow rate of liquid adhering to the surface. The second splashing mechanism observed in our experiments was sheet formation due to lamella instability, a phenomenon well known in liquid jet application to a flat moving surface [4]. Depending on operating conditions, sheet can form in a stable or unstable manner (Fig. 3c and 3d, respectively). Also, sheet formation can be continuous or intermittent (alternating periods of sheet formation and direct deposition, marked by squares in Fig. 6).

The mechanism of intermittent sheet formation (i.e. lamella detachment) was discussed by Guo and Green [5]. According to the authors, the transition between direct deposition and lamella splash (and vice versa) can be induced by flow disturbances such as air bubbles trapped in liquid jets, or small liquid droplets splattered ahead of the jet impingement zone that destabilize the lamella when covered by deposited liquid film. Our experiments confirm that this is also true for liquid deposition on a rotating cylinder (Fig. 7). In most cases, a metastable deposition regime induced by a flow disturbance lasted for a relatively long time (up to a few seconds), until a new disturbance caused the liquid flow to revert to the previous regime, resulting in perpetual splash/no-splash alterations.

Fig. 7. Transition between direct deposition and lamella splashing induced by a flow disturbance ($v_s = 2.40$ m/s, $v_j = 9.23$ m/s, $\varphi = 30^\circ$, $Re = 249$, $We = 2.96 \cdot 10^3$)

As shown in Fig. 6, the Reynolds number boundary between splash and deposition, Re_{CR} , (i.e., the critical Reynolds number) reaches the maximum value of $Re_{CR} = 270 \dots 330$, which is slightly above the threshold reported by [4] for the same roughness ratio $\varepsilon/D = 0.015$ ($Re_{CR} \approx 250$). Nevertheless, certain operating regimes in our experiments resulted in splash occurrence at Reynolds numbers as low as $Re = 100$. This, besides the general complexity of deposition mechanisms distribution presented in Fig. 6, indicates that Re_{CR}

changes in a nonlinear manner with respect to α , thus requiring further dimensional and dimensionless analysis.

For a more detailed investigation of the effect of operating parameters on transition between direct deposition and splashing regime, splash onset curves will be presented in dimensional (Fig. 8) and dimensionless (Fig. 9) terms. In Fig. 8, critical jet velocity for onset of liquid splashing by rim instability-induced droplet shedding (index R) and by lamella instability (index L) is shown as a function of cylinder surface velocity v_s and impingement position φ . The highest jet velocity for the onset of both splashing mechanisms is required in the case of a stationary cylinder surface.

Fig. 8. Critical jet velocity for splash onset (mechanism: R...rim instability, L...lamella instability)

When the cylinder starts to rotate, the critical value of v_j can be seen to decrease until reaching a minimum in the range of $v_s = 5 \dots 7$ m/s. As v_s increases beyond this range, the critical value of v_j also begins to slowly increase. From transition curves presented in Fig. 8 it is evident that the onset of splashing by the rim instability mechanism occurs at 30%...50% of critical jet velocity for lamella instability splashing. Jet impingement position φ also affects the transition from direct deposition to splash, with splash threshold velocity increasing when φ is increased from 15° to 30° . Reduced sensitivity to splashing at higher φ settings is the consequence of increased cylinder surface inclination

in the impingement zone that has been demonstrated to dampen the intensity of splashing in our past research [21].

Dimensional representation of liquid deposition parameters as in Fig. 8 can be very informative about the particular experiment. Nevertheless, dimensionless presentation of transition between deposition regimes (Fig. 9) is also of great importance since it allows for scaling of experimental setup and comparison with other studies. As can be seen from Fig. 9, critical Reynolds number for splash onset is nearly independent of the relative impingement angle for $\alpha < 45^\circ$ for both rim and lamella instability splashing mechanism ($Re_{CR,R}$ and $Re_{CR,L}$, respectively). However, a further increase of α causes to increase exponentially as the theoretical limit of $\alpha = 90^\circ$ is approached. Similar $Re_{CR,L}$ behavior was also observed by Keshavarz et al. [4] for impingement on a flat moving surface.

Fig. 9. Critical Reynolds number for splash onset (mechanism: R...rim instability, L...lamella instability)

In our experiments, $Re_{CR,L} \approx 100$ was observed near $\alpha = 0^\circ$, while Keshavarz et al. [4] determined a corresponding value of $Re_{CR,L} \approx 250$ for $\varepsilon/D = 0.015$ (surface roughness ratio equivalent to our experiments), $Re_{CR,L} \approx 350$ for smooth surface ($\varepsilon/D = 0$) and $Re_{CR,L} \approx 65$ for $\varepsilon/D = 0.21$, meaning that $Re_{CR,L}$ drops very rapidly with surface roughness. Also, jet diameter D in our study was approximately 4 times larger and impinging on an already wetted surface, raising a question whether an equivalent ε/D ratio results in comparable surface conditions to those in the study by Keshavarz et al. [4]. A direct comparison of critical Reynolds numbers by both studies is thus not viable, although the order of

magnitude of $Re_{CR,L}$ is the same. Note that splashing by rim instability was only encountered in our experiments, with critical Reynolds number $Re_{CR,R} = 0.5...0.6 Re_{CR,L}$ for low α angles. If a liquid jet impinges on a horizontal moving surface wider than the expected width of deposited film, liquid rim originating from hydraulic jump remains attached to the surface (provided that lamella instability is not present), thus preventing liquid loss by droplet shedding.

Another observation that can be made with regard to Fig. 9 is an insignificant effect of the impingement position (angle φ) on values of $Re_{CR,R}$ and $Re_{CR,L}$, which is why corresponding linear piecewise curves shown in Fig. 9 were fitted to combined (α, Re_{CR}) data points for $\varphi = 15^\circ$ and $\varphi = 30^\circ$. The onset of liquid deflection from the cylinder surface was also not significantly affected by φ , occurring at $\alpha \approx 82^\circ$ and $Re \approx 395$. These findings can be partly explained by the fact that if the value of α (Eq. (7)) is the same for different combinations of geometric and kinematic parameters, there will be a good kinematic similarity between these cases regardless of the value of particular dimensional parameters such as φ .

Apart from splashing avoidance, knowledge of deposited film properties such as width B and thickness δ is also important for characterization of the liquid deposition process. In Fig. 10, dependence of measured liquid film width B (Fig. 3a) on velocity conditions is presented in dimensionless terms. It is evident that film width increases with v_j/v_s ratio for both direct deposition and splashing operating regime, as the cylinder surface must accommodate larger volumetric flow rate of liquid when v_j is increased, thus resulting in a larger film width. In Fig. 10, power law models were fitted to film width data separately for direct deposition and lamella splash regimes (Eqs. (8) and (9), respectively):

$$\frac{B}{D} = 5.589 \left(\frac{v_j}{v_s} \right)^{0.573} \quad (8)$$

$$\frac{B}{D} = 4.993 \left(\frac{v_j}{v_s} \right)^{0.514} \quad (9)$$

Both models have a high coefficient of determination ($r^2 = 0.996$ for direct deposition and $r^2 = 0.960$ for splash regime model). Nevertheless, there are some limitations when extrapolating presented models to very low v_j/v_s ratios, i.e. a slow jet of liquid impinging on a cylinder surface with high circumferential velocity, which is a situation typical for mineral melt impingement on spinner wheels of stone wool spinners [7][9]. In this case, Eqs. (8) and (9) would predict $B/D < 1$, but in reality B would still be slightly higher than D .

Fig. 10. Relation between characteristic velocities and deposited liquid film width

If the liquid film thickness δ is assumed to be independent of the deposition regime, the fraction of liquid lost by splash can be assumed to be proportional to the reduction of melt film width due to splashing. From Fig. 10 and Eqs. (8) and (9), it can be estimated that approximately 15% of liquid is lost (i.e. not deposited to cylinder surface) due to the onset of lamella splashing. For rim instability splashing, the fraction of lost liquid is lower (likely below 10%), but there is insufficient measurement data for a reliable estimate. δ can now be calculated from the flow rate of liquid deposited in the film (Q_{DEP}). $Q_{\text{DEP}} = Q$ and $Q_{\text{DEP}} = 0.85Q$ (Q is calculated by Eq. (3)) will be assumed for direct deposition and lamella splashing regime, respectively (operating points with rim instability splashing will be disregarded in analysis). When assuming uniform liquid film thickness, deposited flow rate is also equal to $Q_{\text{DEP}} = v_s \cdot \delta \cdot B$, where the product $\delta \cdot B$ is the cross-section area of the film. δ can now be expressed as $\delta = Q_{\text{DEP}} / (v_s \cdot B)$. Dependence of δ/D ratio on v_j/v_s ratio is shown in Fig. 11, where the following power law model was fitted to experimental data ($r^2 = 0.975$):

$$\frac{\delta}{D} = 0.139 \left(\frac{v_j}{v_s} \right)^{0.446} \quad (10)$$

Fig. 11. Relation between characteristic velocities and deposited liquid film thickness

In Fig. 11, one can observe that liquid film thickness grows with increasing ratio of jet to surface velocity. Similarly to B (Eqs. (8) and (9)), the growth of δ is roughly proportional to the square root of v_j/v_s ratio. Nevertheless, caution is required as calculated δ/D value approaches unity ($v_j/v_s \gg 10$), since the inertial force of impinging liquid jet becomes dominant over the viscous force [16], resulting in liquid film thinner and wider than predicted by Eq. (10).

4 Conclusions

An experimental study of liquid jet impingement on a rotating cylindrical free surface has been performed for different impingement positions and velocity conditions using the high-speed imaging technique. On a fundamental level, our results are consistent with existing studies for jet impingement on flat surfaces, although the lubrication mechanism is significantly more complex.

The main conclusions of the present paper are:

1. Liquid jet deposition on rotation cylinder surface generates a wider variety of possible outcomes than deposition on a flat, horizontally translating surface. Phenomena unique to impingement on a rotating curved surface are splashing by rim instability-induced droplet shedding, as well as jet deflection and formation of stripes on a pre-wetted substrate. On the other hand, flow occurrences such as lamella detachment and alternating splash/direct deposition regime due to jet flow disturbances has also been observed in flat surface impingement setups. The exact mechanism for triggering the latter intermittent regimes is yet unknown and

deserves further attention, as elimination of the splash phase could allow for stable direct deposition at significantly larger flow rates and surface velocities.

2. Dimensionless formulations for Reynolds number and relative impingement angle α in the moving surface frame of reference effectively include the significant kinematic and rheological properties of the deposition process. The inhibiting effect of α on the onset of splashing is only significant for applications where cylinder surface velocity is much higher than impinging jet velocity. To improve the understanding of splashing onset and extend boundaries of direct deposition, further research should consider effects of non-circular jets, non-Newtonian rheology and cylinder radius on validity of present formulations.
3. Liquid film width and thickness is roughly proportional to the square root of the ratio of jet to surface velocity, and can be accurately modelled by power-law fitting to experimental data. Nevertheless, the film width can be significantly reduced with the onset of lamella splashing.

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Data availability statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Nomenclature

A	liquid jet cross-section, m^2
B	liquid film width, m
D	jet diameter at impingement point, m
D_0	nozzle inner diameter, m
f	cylinder rotational speed, Hz
L	cylinder length, m
Q	volume flow rate of supplied liquid, m^3/s
Q_{DEP}	volume flow rate of liquid deposited on cylinder surface, m^3/s
q	mass flow rate of supplied liquid, kg/s
R	cylinder radius, m
r	coefficient of determination, -
Re	Reynolds number, -

Re_{CR}	Critical Reynolds number for deposition-splash transition, -
t	time, s
v_{j0}	mean jet velocity at nozzle exit, m/s
v_j	jet velocity at impingement point, m/s
v_R	relative jet impingement velocity magnitude, m/s
v_s	cylinder surface circumferential velocity, m/s
$\mathbf{v}$	velocity vectors, m/s
We	Weber number, -
α	jet impingement angle relative to the surface normal, in surface frame of reference, °
β	angle between jet velocity vectors of absolute and relative reference frame, °
δ	liquid film thickness, m
ε	surface roughness height, m
μ	liquid viscosity, Pa s
ρ	liquid density, kg/m ³
σ	liquid surface tension, N/m
φ	liquid impingement position angle, °

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