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Evaluation of several approaches for deriving weights in fuzzy group analytic hierarchy process

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Abstract
The paper discusses fuzzy group analytic hierarchy process. In the literature there are found several approaches for determining weights either directly from the individual judgments or via group comparison matrix. However, the quality of the group weights was not studied yet. In the paper we propose two new measures for the evaluation of the group weights that are adapted from classical analytic hierarchy process. We selected six approaches for deriving weights and use them in an application from the literature. We evaluated the gained weights. Our results show that the weights derived by the most popular extent analysis method are questionable in their reliability.

Keywords: decision making models; fuzzy analytic hierarchy process; triangular fuzzy numbers; soft consensus; TFNGMDEA; extent analysis method

Introduction

Analytic hierarchy process (AHP) is a widely known multi-criteria decision-making tool for evaluating the decision weights, developed by Saaty (1980). The problems today are comprehensive and complex and one decision maker alone can hardly make reliable decisions. Therefore, group decision making is important part of decision support systems. To deal with uncertainty and vagueness that are often a part of human thinking and estimations fuzzy set theory (Zadeh, 1965) can be a useful tool. Fuzzy group AHP methods generally consist of two parts. In the first part individual judgments are aggregated into group judgments and in the second part the weights are derived from the group comparison matrix. Among many fuzzy AHP methods for deriving weights from fuzzy comparison matrix (Deng, 1999; Grošelj & Zadnik Stirn, 2015; Grošelj & Zadnik Stirn, 2017; Krejčí, 2017; Stam et al., 1996; Van Laarhoven & Pedrycz, 1983; Wang, 2015) the extent analysis method (Chang, 1996) is the most popular. Its main drawback is that it could produce illogical zero weights. Wang et al. (Wang et al., 2008) claim that the weights do not represent the relative importance of compared objects. Another method for determining weights that is often used in applications (Beskese et al., 2015; Kahraman & Cebi, 2009) is fuzzy extension of the geometric mean method that was first introduced by Buckley (1985). An appealing possibility for deriving group weights are consensus and soft consensus reaching models (Grošelj & Zadnik Stirn, 2017; Herrera-Viedma et al., 2014; Srdjevic et al., 2013). The soft consensus is usually an iterative dynamic process which tends to improve the incompatibility between decision makers. The resulting group weights in fuzzy AHP can be fuzzy or crisp.

In the paper, we selected several fuzzy group AHP approaches that produce non-fuzzy crisp weights. The purpose of this study was to evaluate the quality of the derived weights. With this intention in mind we adapted two measures for evaluation from the classical AHP. The evaluation was done on the strategic forest management application from the literature.

In the next section a short recurrence of the fuzzy AHP and the selected group methods is presented. Then, evaluation measures are offered, followed by the example. The results with discussion and the conclusions finish the paper.

Fuzzy analytic hierarchy process

The fuzzy extension of AHP supports the linguistic evaluations such as equally important, slightly more important, moderately more important, strongly more important. Linguistic evaluations efficiently express human perceptions because decision makers or stakeholders, especially less experienced in decision making, often feel more comfortable expressing their judgments as linguistic evaluations rather than as numeric values. Linguistic term is represented by the convex normalized fuzzy set, called fuzzy number, that is defined by the membership function $\mu_A(x)$ which assigns the level of membership in the fuzzy set *A* to the object *x*. Triangular fuzzy numbers (TFNs) (1) are the most widely used. TFN is described by three parameters *l*, *m*, *u* that denote the smallest possible number, the most promising value and the largest possible value, respectively. If $l = m = u$, the TFN is a non-fuzzy crisp value.

$$\mu(x) = \begin{cases} \frac{x-l}{m-l}, & l \le x \le m \\ \frac{u-x}{u-m}, & m \le x \le u \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

In classical AHP scale 1-9 is used for pairwise comparisons. In fuzzy AHP there is not any widely accepted scale but there exist several different possibilities for the triangular fuzzy scale (Ishizaka & Nguyen, 2013). Table 1 presents the fuzzy scale which is adopted from the literature (Chen et al., 2015; Kaya & Kahraman, 2011b; Kutlu & Ekmekçioğlu, 2012), and is also applied in the Example.

Table 1. Triangular fuzzy scale

Linguistic terms	Triangular fuzzy scale
Equally preferable (EQ)	(1, 1, 1)
Slightly preferable (SP)	(1, 1, 3/2)
Fairly preferable (FP)	(1, 3/2, 2)
Very strongly preferable (EP)	(3/2, 2, 5/2)
Absolutely preferable (AP)	(2, 5/2, 3)

The main fuzzy arithmetic operations for two TFNs $\tilde{x}_1 = (l_1, m_1, u_1)$ and $\tilde{x}_2 = (l_2, m_2, u_2)$ applying the extension principle (Zadeh, 1975) are as follows:

$$\begin{aligned} \tilde{x}_1 \oplus \tilde{x}_2 &= \left(l_1 + l_2, m_1 + m_2, u_1 + u_2\right) \\ \tilde{x}_1 \ominus \tilde{x}_2 &= \left(l_1 - u_2, m_1 - m_2, u_1 - l_2\right) \\ \tilde{x}_1 \otimes \tilde{x}_2 &= \left(l_1 l_2, m_1 m_2, u_1 u_2\right) \\ \tilde{x}_1 \oslash \tilde{x}_2 &= \left(\frac{l_1}{u_2}, \frac{m_1}{m_2}, \frac{u_1}{l_2}\right) \end{aligned} \tag{1}$$

Fuzzy comparison matrix is presented in the form $A = (\tilde{a}_{ij})_{n\times n} = \left[\left(l_{ij}, m_{ij}, u_{ij}\right)\right]_{n\times n}$, with $\tilde{a}_{ij} = \tilde{a}_{ji}^{-1} = (1/u_{ij}, 1/m_{ij}, 1/l_{ij})$ for $i, j = 1,...,n$.

Group fuzzy AHP

Let $A^{group} = \left(a_{ij}^{group}\right),\ i, j = 1,...,n$ be a comparison matrix of the TFN judgments $\tilde{a}_{ij}^{group} = \left(l_{ij}^{group}, m_{ij}^{group}, u_{ij}^{group}\right)$. According to the literature (Chang et al., 2011; Chang et al., 2009; Chen et al., 2015; Larimian et al., 2013) the most common way to aggregate *m* individual fuzzy comparison matrices into group fuzzy comparison matrix $A^{group} = (\tilde{a}_{ij}^{group})_{n\times n} = \left[\left(l_{ij}^{group}, m_{ij}^{group}, u_{ij}^{group}\right)\right]_{n\times n}$ is as follows:

$$l_{ij}^{group} = \min_{k=1,2,...,\mathrm{m}} \left\{a_{ij}^{(k)}\right\}$$
$$m_{ij}^{group} = \left(\prod_{k=1}^{m} a_{ij}^{(k)}\right)^{1/\mathrm{m}} \tag{2}$$
$$u_{ij}^{group} = \max_{k=1,2,...,\mathrm{m}} \left\{a_{ij}^{(k)}\right\}$$

Another possibility is to take geometric mean of individual fuzzy comparison matrices (Beskese et al., 2015; Meixner, 2009).

Modified extent method

The extent analysis method (Chang, 1996) was one of the first methods presented in the literature for determining weights from a fuzzy pairwise comparison matrix. Later it was improved (Heo et al., 2010; Heo et al., 2012; Wang & Elhag, 2006; Wang et al., 2008; Zhu et al., 1999) into modified extent analysis method, which is nowadays one of the most popular methods. In the first stage the normalized synthetic extents with respect to the *i*-th object are defined as

$$S_i = \left(\frac{\sum_{j=1}^{n} l_{ij}}{\sum_{j=1}^{n} l_{ij} + \sum_{k=1,k\neq i}^{n} \sum_{j=1}^{n} u_{kj}}, \frac{\sum_{j=1}^{n} m_{ij}}{\sum_{k=1}^{n}\sum_{j=1}^{n} m_{kj}}, \frac{\sum_{j=1}^{n} u_{ij}}{\sum_{j=1}^{n} u_{ij} + \sum_{k=1,k\neq i}^{n} \sum_{j=1}^{n} l_{kj}}\right),\ i = 1,...,n \tag{3}$$

applying fuzzy arithmetic operations. In the second stage the normalized synthetic extents are compared. The degree of possibility of $S_i = (l_i, m_i, u_i) \geq S_j = (l_j, m_j, u_j)$ is defined as

$$V(S_i \geq S_j) = \begin{cases} 1, & \text{if } \mathrm{m_i} \geq \mathrm{m_j} \\ 0, & \text{if } \mathrm{l_j} \geq \mathrm{u_i} \\ \frac{l_j - u_i}{(m_i - u_i) - (m_j - l_j)}, & \text{otherwise} \end{cases} \tag{4}$$

and the degree of possibility of a fuzzy number *S* to be greater than *k* fuzzy numbers $S_i,\ i = 1,...,k$ is defined as

$$V(S \geq S_1, S_2, ..., S_k) = \min_{i=1,...,k} V(S \geq S_i) \tag{5}$$

Consequently, the non-fuzzy vector of weights is obtained as

$$W' = (d'(A_1), d'(A_2), ..., d'(A_n)), \tag{6}$$

denoting $d'(A_i) = \min_{k=1,...,n;k\neq i} V(S_i \geq S_k)$. Finally, vector W' is normalized to a vector of weights *W*.

Fuzzy extension of the geometric mean method

First, geometric mean of each row of fuzzy comparison matrix is calculated. It is normalized in the second step. In the third step the centroid method based on the center of the gravity as one of the most commonly used method for the defuzzification of fuzzy weights (Opricovic & Tzeng, 2003) is applied:

$$
\begin{aligned}
\tilde{r}_i &= \sqrt[n]{\tilde{a}_{i1} \otimes \tilde{a}_{i2} \otimes \cdots \otimes \tilde{a}_{in}} \\
\tilde{w}_i &= \tilde{r}_i \,\%\left(\tilde{r}_1 \oplus \tilde{r}_2 \oplus \cdots \oplus \tilde{r}_n\right) \\
w_i &= \frac{\tilde{w}_i}{\sum_{j=1}^{n} \tilde{w}_j} = \frac{w_{il} + w_{im} + w_{iu}}{\sum_{j=1}^{n} \tilde{w}_j}
\end{aligned}
\tag{7}
$$

Soft consensus model

Soft consensus model (Grošelj & Zadnik, 2017) is an extension of the peer-to-peer consensus reaching model (Dong & Cooper, 2016) from classical AHP. It is based on the iteration process which aggregates individual judgments into group judgments. In each iteration only the fuzzy comparison matrices of the most incompatible decision makers are changed in the way that they become more similar. Their similarity is measured by individual fuzzy consensus index (IFCI)

$$
IFCI_{pq} = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{l_{ij}^{(p)} u_{ji}^{(\mathrm{q})} + 4 m_{ij}^{(p)} m_{ji}^{(\mathrm{q})} + u_{ij}^{(p)} l_{ji}^{(\mathrm{q})}}{6} \tag{7}
$$

In each iteration the new individual comparison matrices are defined as follows:

$$
a_{ij}^{(k)t+1} = \begin{cases}
\left(a_{ij}^{(k)t}\right)^{\alpha_k^t} \left(a_{ij}^{(\mathrm{q})t}\right)^{1-\alpha_k^t}, & k = p \\
\left(a_{ij}^{(k)t}\right)^{\alpha_k^t} \left(a_{ij}^{(\mathrm{p})t}\right)^{1-\alpha_k^t}, & k = q \\
a_{ij}^{(k)t}, & k \neq p, q
\end{cases} , \tag{8}
$$

New fuzzy comparison matrices of decision makers DM_p and DM_q are composed of their fuzzy comparison matrices from previous iteration, where the portion of DM's judgments preservation depends on the weight α_p^t:

$$
\alpha_p^t = 1 - \frac{\sum_{i=1, i \neq p,q}^{m} IFCI_{pi}}{2\left(\sum_{i=1, i \neq p,q}^{m} IFCI_{pi} + \sum_{i=1, i \neq p,q}^{m} IFCI_{qi}\right)} \tag{9}
$$

The iteration process stops when IFCI between all pairs of decision makers are below the assigned threshold which is in this paper set to 1.01.

TFNGMDEA method

TFNGMDEA method (Grošelj & Zadnik Stirn, 2015) is an expansion of WGMDEA method (Grošelj et al., 2011) for deriving group weights in classical AHP. It consists of *n* linear programs, which originate from data envelopment analysis:

$$\begin{aligned} &\max w_0 = \sum_{j=1}^{n} \left(l_{0j}^{group} \cdot m_{0j}^{group} \cdot u_{0j}^{group} \right)^{1/3} x_j \\ &subject\ to\ \sum_{j=1}^{n} \left(\sum_{i=1}^{n} \left(l_{ij}^{group} \cdot m_{ij}^{group} \cdot u_{ij}^{group} \right)^{1/3} \right) x_j = 1, \\ &\qquad \sum_{j=1}^{n} \left(l_{ij}^{group} \cdot m_{ij}^{group} \cdot u_{ij}^{group} \right)^{1/3} x_j \geq n x_i,\ \ i = 1,\dots,n, \\ &\qquad x_j \geq 0,\ \ j = 1,\dots,n. \end{aligned} \tag{10}$$

Measures for evaluating group fuzzy AHP methods

To assess the quality of the vectors of weights suitable measures for evaluation are needed. The most known measures are minimum violations criterion (MV) and the generalized Euclidean distance (ED) (Srdjevic, 2005) and their generalizations to the group case (GMV and GED) (Grošelj et al., 2015). A smaller value denotes better estimate for both measures. The MV counts all violations associated with the order reversals, when the decision maker prefers the *j*th criterion to the *i*th criterion and the *i*th group weight is greater than the *j*th group weight. The GMV averages the MV across all individuals. The GED measures the distances between the judgments in comparison matrices of all individuals and the related ratios of priorities from the group vector of weights and averages across all decision makers.

Here we generalized these measures to the fuzzy group case. We took into account that the individual pairwise comparisons are fuzzy values, while the group vectors of weights are crisp.

$$fGMV = \frac{1}{m} \sum_{k=1}^{m} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} I_{ij},\ I_{ij} = \begin{cases} 1 & if \quad w_i > w_j \quad and \quad l_{ij}^{(k)} < 1, \\ 1 & if \quad w_i < w_j \quad and \quad u_{ij}^{(k)} > 1, \\ 0.5 & if \quad w_i = w_j \quad and \quad a_{ij} \neq (1,1,1), \\ 0.5 & if \quad w_i \neq w_j \quad and \quad a_{ij} = (1,1,1), \\ 0 & otherwise \end{cases} \tag{11}$$

For fGED we combined GED and the distance measurement between two fuzzy numbers $d(\tilde{x}_1, \tilde{x}_2) = \sqrt{\frac{1}{3}\left((l_1 - l_2)^2 + (m_1 - m_2)^2 + (u_1 - u_2)^2 \right)}$ as follows:

$$fGED = \frac{1}{m} \sum_{k=1}^{m} \sqrt{ \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{3} \left(\left(l_{ij}^{(k)} - \frac{w_i}{w_j} \right)^2 + \left(m_{ij}^{(k)} - \frac{w_i}{w_j} \right)^2 + \left(u_{ij}^{(k)} - \frac{w_i}{w_j} \right)^2 \right) }. \tag{12}$$

Example

We selected a strategic forest management application from (Kaya & Kahraman, 2011a) . The authors provided pairwise comparison matrices of all decision makers. The goal of the study was to evaluate the alternative watershed district of Istanbul, Turkey regarding forestation and forest preservation. For evaluating six forestation districts (Terkos, Buyukcekmece, Sazlıdere, Pabucdere Omerli, and Darlık), and six criteria were defined. The criteria areWatershed preservation (C1), Soil erosion prevention (C2), Cost efficiency (C3), Land availability (C4), Social acceptability (C5), and Political acceptability (C6). Here we focused only on the comparison of criteria and determination of their weights.

There are many different methods and models in the literature that are designed for different types of problems and the decision maker must know the methods well enough to choose the right one. When the right method is chosen, it is important that the decision maker really knows the method in details so that it can be used correctly. It is also necessary to follow the development of the method in the literature. Further, **s**uch complex decisions also demand for the inclusion of various stakeholders. In the group decision making it is important to know how to aggregate opinions of several stakeholders or decision makers into group opinion. However, the authors of the study violated several of the aforementioned principles. First, they applied arithmetic mean to aggregate individual pairwise comparisons into group comparisons, which does not preserve the reciprocal property. Next, they used the old version of the Extent analysis method to derive weights of criteria. It has been proven that the results of this method are not reliable (Wang & Elhag, 2006). Finally, they made a typing error that influenced the final results. So, the results of their study are wrong.

Results

In our study we selected six different approaches for deriving crisp group weights. Geometric mean method and modified extent analysis method require group comparison matrix. We combined them with the two presented aggregation techniques, the first one presented by equation (3) (named MIN, MAX), and the geometric mean of the individual judgments (named GEO). The fifth approach is the soft consensus model for obtaining the group comparison matrix, combined by the geometric mean method for deriving weights. The sixth approach is TFNGMDEA method. We applied the selected methods to the initial data of the application (Kaya & Kahraman, 2011a)and evaluated the results, which are presented in Table 2.

Table 2. The weights of the criteria derived from six selected methods and the results of the evaluation by fGMV and fGED.

	Geometric mean method GEO	Geometric mean method MIN, MAX	Modified extent method GEO	Modified extent method MIN, MAX	Soft consensus model	TFNGMDEA
Watershed preservation (C1)	0.222	0.231	0.366	0.250	0.222	0.220
Soil erosion prevention (C2)	0.187	0.191	0.253	0.204	0.187	0.186
Cost efficiency (C3)	0.139	0.138	0.061	0.122	0.139	0.140
Land availability (C4)	0.143	0.141	0.078	0.133	0.143	0.144
Social acceptability (C5)	0.153	0.151	0.122	0.148	0.153	0.153
Political acceptability (C6)	0.155	0.147	0.120	0.143	0.155	0.157
fGMV	10	10	10	10	10	10
fGED	1.585	1.610	7.090	1.795	1.585	1.586

The results show very similar weights for the first and the fifth method (the weights are equal on the first three decimal places) and also for the sixth method. The first and the fifth methods produce the group comparison matrix (the geometric mean of individual judgments by the first method and the soft consensus by the second method) and apply the same method for deriving weights. The ranking of the criteria is the same for the first, fifth and sixth method, and differs on the third and the fourth place regarding second, third and fourth method. However, the results of the fGMV measure are equal for all methods. We can conclude that in our case the fGMV is too weak measure for the evaluation of methods. Similar conclusions were reached in the classical group AHP (Grošelj et al., 2015). The results of the fGED show that the most similar methods, namely first, fifth and sixth have the best evaluations. They are followed by the second and the fourth method, while the third method has the worst evaluation. The first and the second method use different approaches for obtaining the group comparison matrix. The first method uses geometric mean of individual judgments, while the second uses minimum of the individual lower bounds for the group lower bound and maximum of the individual upper bounds for the group upper bound. If the individual judgments are inhomogeneous, the group judgments can have very wide support resulting in less accurate weights. So, the worse evaluation of the second method was expected. However, the results of the third and the fourth methods are illogical. Both methods are based on the modified extent analysis and differ in approaches for obtaining group comparison matrix as the first and the second methods. According to the above reasoning the evaluation of the third method should be better than that of the fourth method, what is not the case. We can conclude that the extent analysis is a problematic method and that the derived weights really do not represent the importance of criteria as claimed in (Wang et al., 2008).

Conclusions

Fuzzy group AHP method is often used in the applications. There are many known approaches for deriving weights. However, the approaches are not critically evaluated. Our study is the first step in this direction. Our results confirmed that the extent method that is most popular and widely used in applications is a problematic method. In the future research more extensive evaluations should be performed, including more methods. The evaluations should be applied on data from the real world applications, and also on the theoretically modelled data.

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