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Toward a solid-state thermal diode for room-temperature magnetocaloric energy conversion

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Thermal control elements, i.e., thermal diodes, switches, and regulators, can control the heat flow in an analogous way to how electronic devices control electrical currents. In particular, a thermal diode allows a larger heat flux in one direction than in the other. This has aroused the interest of researchers working on the thermal management of electronics, refrigeration, and energy conversion. Solid-state thermal diodes are attractive because they are silent, reliable, lightweight, and durable. While some solid-state thermal diodes have been developed at the nano- and microscales, the leap to the macroscale has yet to be made. A macroscale thermal diode would play a crucial role in the future development of applications related to caloric refrigeration and heat pumping. Additionally, the temperature changes of caloric materials (due to the caloric effect) are ideal for testing these thermal devices. This paper aims to numerically evaluate the influence of a macroscopic solid-state thermal diode in a magnetocaloric refrigeration device under transient and quasi-steady-state conditions. Materials with different temperature-dependent properties were analyzed and the most promising ones were selected for the operating range of a magnetocaloric device (290–296 K). The highest achieved magnetocaloric thermal rectification ratio under transient conditions was up to 295-times higher than with quasi-steady-state operation. This shows that transient operation should be considered for future progress with this technology.

Keywords: thermal rectification, thermal diode, thermal rectifier, thermal control element, heat transfer, magnetocaloric, energy conversion, refrigeration

Nomenclature

		Subscripts	
c_p	specific heat capacity [J kg ⁻¹ K ⁻¹]	demag	demagnetization (stage III in the thermodynamic cycle)
h	convective heat-transfer coefficient [W m ⁻² K ⁻¹]	diode	thermal diode
k	thermal conductivity [W m ⁻¹ K ⁻¹]	fwd	forward direction
$\dot{Q}$	heat flux [W]	i	discretization in time (from 1 to N)
R	rectification ratio []	mag	magnetization (stage I in the thermodynamic cycle)
ρ	density [kg m ⁻³]	mcm	magnetocaloric material
T	temperature [K]	n	discretization in space (from 1 to m)
t	time [s]	rev	reverse direction
x, L	length [m]		

Abbreviations	
TD	thermal diode
SS	solid state
ACR	active caloric regeneration
mcm	magnetocaloric material
MC	magnetocaloric

sink	heat sink
source	heat source

I. INTRODUCTION

Caloric refrigeration is seen as one of the promising alternatives to vapor-compression refrigeration technology^{1–3}. This technology is based on exploiting the so-called caloric effect, where the temperature of a caloric material changes with a changing external parameter: i) magnetic field for a magnetocaloric material^{4–7}; ii) electric field for an electrocaloric material^{8–11}; iii) applied stress for an elastocaloric material^{12–14}; iv) applied pressure for a barocaloric material^{15,16} or; v) a combination of some or all of them for a multicaloric material^{17–19} under adiabatic conditions. A caloric refrigeration or heat-pump system typically consists of a caloric material, heat source, heat sink, and heat-transfer medium, and its thermodynamic cycle comprises the following four stages:

- I. Application of the external field under adiabatic conditions, which leads to an increase in the temperature of the caloric material.
- II. Heat transfer from the caloric material to the heat sink while the external field is being applied.
- III. Removal of the external field under adiabatic conditions, which leads to a decrease in the temperature of the caloric material.
- IV. Heat transfer from the heat source to the caloric material when no external field is applied.

The operating frequency defines the number of completed thermodynamic cycles per unit of time. Most caloric devices use a fluidic heat-transfer medium that oscillates through a porous active caloric regenerator (ACR), first proposed by Barclay and Steyert in 1981²⁰ for magnetocalorics and later used in electrocalorics²¹, elastocalorics¹⁴, and barocalorics²². The regeneration of heat inside the ACR enables larger temperature spans than the adiabatic temperature change of the caloric material. Unfortunately, an ACR only works efficiently at low fluid velocities, which limits the operating frequency and the cooling-power density of a caloric device⁵. The performance is also reduced by the dead volume, viscous losses, carryover leakage and fluid maldistribution within the ACR⁵. An alternative to the ACR is a thermal control element, i.e., a thermal switch, thermal regulator, or thermal diode (TD) (Fig. 1), which can rectify, switch on/off or change the direction of the heat flux between two surfaces/bodies at different temperatures.

FIG. 1. Operation of thermal control elements. (Reprinted from Renewable and Sustainable Energy Reviews, Vol 118, Kliner K., Kitanovski A., Thermal control elements for caloric energy conversion, Copyright (2020), with permission from Elsevier).

An example of this is when the caloric material is embodied within a bipartite TD, of which one part serves for the thermal connection between the caloric material and the heat source, and the other as the thermal connection between the caloric material and the heat sink (Fig. 2). To efficiently transfer the heat from the heat source to the heat sink in the caloric device, the TD has to operate as follows. When the caloric material is heated as a result of the caloric effect (i.e., stage II, as referred to above), the TD must allow the heat to flow only from the caloric material toward the heat sink, and simultaneously prevent any heat flux from the caloric material to the heat source. In contrast, when the caloric material is cooled as a result of the caloric effect (i.e., stage IV, as referred to above), the TD must allow the heat to flow only from the heat source to the caloric material, and simultaneously prevent any heat flux from the heat sink to the caloric material.

FIG. 2. Schematics of implementing a TD in a caloric device.

The application of thermal control elements in a caloric device would mean higher operating frequencies compared to an ACR, which would increase the cooling-power density and the compactness of the device^{23–25}. The idea was proposed for different caloric technologies by Basiulis *et al.*²⁶, Mathur and Mishchenko²⁷ and Epstein *et al.*²⁸. In a recently published review paper, Klinar and Kitanovski²⁹ reported on the application of thermal control elements in caloric devices. Solid-state (SS) TDs could be an important solution for magneto- and electrocaloric devices, especially because of their fully static and also passive operation, without the need for any external work input (i.e., electric, magnetic, mechanical)³⁰. The implementation of a bipartite SS TD could also mean the potential to increase the cooling-power density of a device in a similar way to thermal switches²⁹. However, at the present stage of development, the TDs are not yet ready for implementation into a caloric device. The main reasons for this are the missing information on candidate materials, a lack of comprehensive simulations and a lack of demonstration experiments.

Of the different types of TD, only a TD based on the heat-pipe principle was tested experimentally for caloric refrigeration by Sato³¹ (electrocaloric), Maier *et al.*³² (magnetocaloric) and Bartholomé *et al.*³³ (elastocaloric). The numerical modeling was only performed for thermal switches²⁹; however, the lumped-capacitance numerical model of Hess *et al.*³⁴ also considered TDs in a cascaded caloric device. The model considered idealized TDs, which do not allow heat flux in the reverse direction and do not have any thermal capacity.

The objective of our work was to evaluate the thermal properties of materials that can be used for a SS TD applied in a magnetocaloric device. For the purposes of this study, a numerical model based on the transient macroscopic heat transfer was developed. This represented the basis for a numerical analysis, which was performed for different operating conditions of the magnetocaloric device, consisting of TDs with different material properties.

II. SOLID-STATE THERMAL DIODE

A TD exhibits an asymmetrical heat transfer in the forwards and backwards directions³⁵ (Fig. 1). Thermal rectification has been observed in bulk materials on the macroscale^{36–46}, with the thermal conductivity in the material having a spatial and a temperature dependence, as shown by Go and Sen³⁶. The most straightforward way to achieve anisotropic thermal conductivity at the macroscale is to join two different materials with an inverse temperature-dependent thermal conductivity^{37,38}. Thermal rectification also occurs in a composite of two homogeneous materials where only one of them has a temperature-dependent thermal conductivity³⁸. The thermal conductivity in these materials depends on the absolute temperature; however, the heat flux in a composite TD depends on the temperature gradient across the TD (Fig. 2).

The rectification ratio R determines the performance of the TD with respect to rectifying the heat and can be calculated in different ways^{37,44,47}. The aim is to compare the heat flux through the TD in the forward $\dot{Q}_{\text{fwd}}$ and the reverse directions $\dot{Q}_{\text{rev}}$. The most common expression used to determine R when $\dot{Q}_{\text{fwd}} > \dot{Q}_{\text{rev}}$ is presented in Eq. (1)^{38,39,41–43,46,48,49}.

$$R = \left| \frac{\dot{Q}_{\text{fwd}}}{\dot{Q}_{\text{rev}}} \right| \quad (1)$$

The most promising candidate materials for such a TD are solid-solid phase-change materials, for example, vanadium oxide⁵⁰, polyethylene nanofibers^{51,52}, and silver chalcogenides⁵³. However, at this stage no material that would exhibit a very

high and discontinuous change of thermal conductivity in the required temperature interval was found in the literature. Our selection includes the best-case scenarios of hypothetical solid-solid phase-change materials.

The rectification ratio R was chosen as the main target parameter for our analysis of a TD in a caloric device, because it directly influences its energy efficiency. As pointed out in the introduction, the TD must establish a one-way heat flow between the heat source and the heat sink. Ideally, the TD would have an infinite R , meaning there is no heat flux in the reverse direction ($\dot{Q}_{\text{rev}} = 0$). Anytime $\dot{Q}_{\text{rev}} > 0$, undesired heat flux (heat loss) decreases the performance of the caloric device.

III. NUMERICAL MODEL

The thermal rectification in a SS bi-layered composite TD under steady-state conditions has typically been solved by numerical modeling^{35,39,41–43}, but sometimes also analytically for specific cases of planar^{37,40,44–46} and cylindrical⁴⁹ structures. Only Herrera *et al.*⁵⁶ investigated the transient behavior of such types of TD, showing an increased R before the steady state was reached in the diode. We now want to analyze the performance of similar diodes under transient conditions when integrated into a magnetocaloric device, accounting for the temperature changes of the magnetocaloric material during magnetization or demagnetization.

A magnetocaloric refrigeration device using a TD instead of an ACR is presented in Fig. 3b, where the magnetocaloric material (denoted as mcm) is between two parts of the TD, denoted as TD1 and TD2. A heat source and a heat sink are attached to the end of this device. Each part of the TD consists of two blocks of material, A and B, which lead to rectification effects³³. Since the goal is to test the possibility of implementing TDs in a magnetocaloric device, they were first tested in the simplified configuration shown in Fig. 3a. The magnetocaloric effect was simplified by considering the interface temperatures as the boundary conditions between the TD and the magnetocaloric material, which led to two different situations for each part of the TD (Fig. 3a₁, 3a₂, and 3a₃, 3a₄). The boundary temperatures represent the quasi-steady state of the magnetocaloric device. The arrows in Fig. 3b show the direction and intensity of the heat fluxes $\dot{Q}$. On the one hand, the heat sink was considered to be held at $T_{\text{sink}}=294.5$ K, while the heat source at $T_{\text{source}}=291.5$ K. On the other hand, we also account for an adiabatic temperature change of 2.5 K for the magnetocaloric material between $T_{\text{mcm,demag}}=290.5$ K (after demagnetization) and $T_{\text{mcm,mag}}=295.5$ K (after magnetization). The magnetocaloric material and the two heat exchangers were set as large thermal reservoirs so that the temperature at all the interfaces remained constant during the simulations.

FIG. 3. a) 3a₁-3a₄ are the four different situations of TD parts, TD1 and TD2, evaluated by the numerical model 1. b) Schematics of the investigated magnetocaloric device with implemented TD parts, TD1 and TD2 (numerical model 2).

TDs are usually tested^{47,57} by observing the heat flux through the diode with the boundary temperatures T_1 and T_2 being constant, and only switch sides in order to define the forward vs. reverse heat flux. In our case the situation is different because the temperature span was not the same in the forward and reverse directions across each part of the TD (see Fig. 3b). In addition, the temperature at the side of the heat exchanger was held constant, while the temperature at the interface with the magnetocaloric material was different for the cases after magnetization and demagnetization. It is also possible to define the magnetocaloric thermal rectification ratio, but it is important to note that it cannot be directly compared with the above-mentioned R .

Based on the requirements for the proper operation of this magnetocaloric device, $\dot{Q}_2 > \dot{Q}_1$ after magnetization and $\dot{Q}_3 > \dot{Q}_4$ after demagnetization are required (see Fig. 3a). Additionally, $\dot{Q}_3 > \dot{Q}_1$ and $\dot{Q}_2 > \dot{Q}_4$ for each part of the TD. The transient thermal rectification, i.e., the temporal heat-flux variation across the system before reaching the quasi-steady-state conditions in the TD, was also evaluated. The four different rectification ratios $R_{\text{mag}}(t)$, $R_{\text{demag}}(t)$, $R_{\text{diode1}}(t)$ and $R_{\text{diode2}}(t)$ were defined as follows

$$R_{\text{mag}}(t) = \left| \frac{\dot{Q}_2(t)}{\dot{Q}_1(t)} \right|, \quad R_{\text{demag}}(t) = \left| \frac{\dot{Q}_3(t)}{\dot{Q}_4(t)} \right|, \quad R_{\text{diode1}}(t) = \left| \frac{\dot{Q}_3(t)}{\dot{Q}_1(t)} \right|, \quad R_{\text{diode2}}(t) = \left| \frac{\dot{Q}_2(t)}{\dot{Q}_4(t)} \right|. \quad (2)$$

This was calculated by considering the heat fluxes in the nodes at the interface between TD1 and the heat source, and between TD2 and the heat sink.

The numerical model consisted of an implicit finite-difference scheme that used Fourier's law of heat conduction (Eq. (S1) in *Supplementary Material*) in a 1D bipartite TD1 and TD2, where the diode materials A and B (see Fig. 3) exhibited a temperature-dependent thermal conductivity $k(T)$. The details of the numerical models and the validation^{38,41} are given in the *Supplementary Material*.

After validation we considered the input parameters applied in the magnetocaloric device: i) the temperature ranges observed in Fig. 3a) and; ii) the data that was modified to meet the required magnitudes of the heat fluxes $\dot{Q}_{1-4}$. To achieve the desired heat fluxes, the thermal conductivity, density, and specific heat capacity of the parts of the TD were varied. The total length of TD1 and TD2 was fixed at 10 mm, with the length of material A varying from 1 to 9 mm and that of material B from 9 to 1 mm. The performance of the different TDs was compared based on the rectification ratios defined by Eq. (2), which had to be greater than 1. The simplified steady-state and transient simulations were performed with the numerical model 1. The best-case scenario was then tested in numerical model 2, which returned the quasi-steady-state temperature profile of a 1D magnetocaloric device (Fig. 3b). This numerical model 2 was additionally used to compare the operation of a magnetocaloric device with and without TDs. The complete procedure for determining the best TD1 and TD2 is shown in Fig. 4.

FIG.4. Process for determining the best candidates for TD1 and TD2 using numerical models 1 and 2.

IV. RESULTS

The search for the best TD was carried out step by step, as shown in Fig. 4. The analysis of the SS TD was divided into the quasi-steady state in the first part and the transient simulations in the second part. In the third part the best TD was implemented in numerical model 2 of a 1D magnetocaloric device for further tests.

Since the idea was to find the simplest composition of a TD that shows a satisfactory R , we started with the composition of the materials A_1 - B_1 (TD1) and A_2 - B_2 (TD2), where A_1 and A_2 had constant thermal conductivities and B_1 and B_2 were temperature dependent. Interestingly, the highest rectification ratios for this arrangement were achieved in the cases where TD1 (Fig. 3b₁ and Fig. 3b₃) and TD2 (Fig. 3b₂ and Fig. 3b₄) consisted only of the materials B_1 and B_2 , respectively. Therefore, all additional simulations were performed for a TD consisting only of material B_1 (TD1) or B_2 (TD2) with a temperature-dependent thermal conductivity function of hypothetical solid-solid phase-change materials. The quasi-steady-state simulations were continued with the analysis of the different thermal conductivity functions of the materials B_1 and B_2 between the operating temperatures 290.5 K and 295.5 K. Some examples of temperature-dependent thermal conductivity functions in hypothetical materials that were implemented in parts of the TD are shown in Fig. 5. The R under quasi-steady-state conditions presented in Table I shows that the majority of the tested functions resulted in no rectification ($R < 1$). This is due to the fact that the minimum (min) and maximum (max) values of the thermal conductivity were too close together to compensate for the heat flux through the diode due to the higher/lower temperature span in the reverse situation. Cases 10 and 11 show an asymmetrical R , which is a consequence of the translated low/high thermal conductivity junction relative to the average evaluated temperature (293 K). The highest rectification ratio $R=2.75$ was achieved in cases 4, 7 and 12, where the minimum thermal conductivity was 1000-times lower than the maximum. The R would be even higher if the ratio between the minimum and maximum thermal conductivity was to be further increased.

FIG. 5. Different functions of the temperature-dependent thermal conductivity of material B₁ (solid line) for TD1 and B₂ (dashed line) used for TD2. For every case the min and max thermal conductivities are also listed. The best results were obtained for cases 4, 7 and 12.

TABLE I. Quasi-steady-state rectification ratios for different temperature dependencies presented in Fig. 5.

Case #	$R_{\text{diode1, steady-state}}$ []	$R_{\text{diode2, steady-state}}$ []	Case #	$R_{\text{diode1, steady-state}}$ []	$R_{\text{diode2, steady-state}}$ []
1	<1	<1	7	2.75	2.75
2	<1	<1	8	<1	<1
3	<1	<1	9	<1	<1
4	2.75	2.75	10	1.27	<1
5	<1	<1	11	<1	1.27
6	<1	<1	12	2.75	2.75

In the second part, the transient simulations were continued, where the TDs consisted only of the temperature-dependent materials B₁ and B₂, whose thermal conductivity functions are shown in Fig. 5 (case 7). Besides the thermal conductivity, also the density and specific heat capacity of the materials play an important role in the transient simulations. Therefore, we

performed an additional series of simulations, testing different combinations and values of the temperature-independent volumetric heat capacities ρc_p .

Since the material properties affect the maximum rectification ratio as well as its time evolution, we calculated the time-integrated rectification ratio $\tilde{R}_{\text{diode}}$, as proposed by Herrera *et al.*⁵⁶. This allowed a comparison of TDs with different densities and specific heat capacities.

$$\tilde{R}_{\text{diode1}} = \int_0^t R_{\text{diode1}}(t) \quad \tilde{R}_{\text{diode2}} = \int_0^t R_{\text{diode2}}(t) \quad (3)$$

Fig. 6 presents the time evolution of heat fluxes $\dot{Q}_1$ and $\dot{Q}_3$, the rectification ratio R_{diode1} of TD1 and the shaded area defines $\tilde{R}_{\text{diode1}}$.

FIG. 6. Example of the time evolution of heat fluxes $\dot{Q}_1$ (solid line) and $\dot{Q}_3$ (dashed line) and the rectification ratio of TD1 R_{diode1} . Time-integrated rectification ratio $\tilde{R}_{\text{diode1}}$ is represented by the gray-shaded area below the gray line.

The time interval t was defined based on the operating frequency of the magnetocaloric device for three different cases. Taking into account that the time required for magnetization (stage I) or demagnetization (stage III) is reported to be 10 ms⁵⁸, this involves 0.49 s for the heat transfer (for each stage II and IV) across the TD at an operating frequency of 1 Hz, 0.04 s at 10 Hz and 0.015 s at 20 Hz. In these tests, the TD consisted of only one material that exhibited a temperature-dependent thermal conductivity (case 7, Fig. 5) and was 10 mm long.

We evaluated the performance of each part of the TD with numerical model 1. Fig. 7 shows the maximum and time-integrated rectification ratios of TD1 for all three frequencies in terms of the volumetric heat capacity ρc_p . The operation of TD2 was similar to TD1 and is therefore not presented. The ideal material (ρc_p) for each part of the TD could be suggested based on the maximum value of $\tilde{R}_{\text{diode}}$ and the junction of both curves and must be optimized for each frequency.

FIG. 7. Time-integrated $\tilde{R}_{\text{diode1}}$ and maximum rectification ratio $R_{\text{diode1,max}}$ of TD1 in terms of volumetric heat capacity, shown for three different operating frequencies.

To achieve the satisfactory operation of the whole magnetocaloric device, $R_{\text{mag}}(t)$ and $R_{\text{demag}}(t)$ must also be considered. Table II shows the best results for two TDs based on the maximum achieved value of all four time-integrated rectification ratios, i.e., $\tilde{R}_{\text{diode1}}$, $\tilde{R}_{\text{diode2}}$, $\tilde{R}_{\text{mag}}$, and $\tilde{R}_{\text{demag}}$ for the three operating frequencies. The time-integrated rectification ratios

$\tilde{R}_{\text{diode1}}$, $\tilde{R}_{\text{diode2}}$ in the table are substantially higher than the time-integrated ratios for the constant quasi-steady-state values $\tilde{R}_{\text{steady-state}}$ of 13475 at 1 Hz, 1100 at 10 Hz, and 413 at 20 Hz.

TABLE II. Most promising results according to the time-integrated rectification ratios presented for different operating frequencies. TDs made only of material B were 10 mm long and exhibited the thermal conductivity presented in case 7 (Fig. 5).

	$\tilde{R}_{\text{diode1}}$ [s]	$\tilde{R}_{\text{diode2}}$ [s]	$\tilde{R}_{\text{mag}}$ [s]	$\tilde{R}_{\text{demag}}$ [s]	$R_{\text{diode1,max}}$ []	$R_{\text{diode2,max}}$ []	ρc_{p_TD1} [J m ⁻³ K ⁻¹]	ρc_{p_TD2} [J m ⁻³ K ⁻¹]
1 Hz	161270	167593	161485	167380	657	811	1.5 x 10 ⁶	1.5 x 10 ⁶
10 Hz	14355	14372	14363	14365	112	143	1.2 x 10 ⁵	1.2 x 10 ⁵
20 Hz	5294	5213	5199	5209	91	102	5 x 10 ⁴	5 x 10 ⁴

Finally, we can consider the operation of a 1D magnetocaloric device with a TD (as shown in Fig. 3b) of the best-case scenario for 20 Hz compared to the operation without a TD using numerical model 2. In the case without a TD, a material with a constant thermal conductivity of 5 W m⁻¹ K⁻¹ was used instead of a TD. A magnetocaloric refrigeration device transfers heat from the heat source to the heat sink. In order to do this, the temperature of TD1 must be lower than that of the heat source, and the temperature of TD2 must be higher than the temperature of the heat sink. Fig. 8 shows the temperature profiles through the device after stage II and after stage IV of the magnetocaloric thermodynamic cycle for the case with (dashed line) and without (solid line) TDs. In the case without the TD the heat propagated from the heated MCM toward both the heat sink and the heat source after stage II and inversely toward the MCM after stage IV. Considering the cyclic operation of the magnetocaloric device, the average temperature of TD1 at the heat source would be too high to absorb heat from the heat source and the average temperature of TD2 at the heat source would be too low to reject heat to the heat sink. This means that such a device without a TD cannot operate, because the heat is not transferred from the heat source to the heat sink. The best case scenario with a TD resulted in a 1.35 K lower average temperature at the heat source ($T_{\text{source}} = 291.3$ K at $x=0$) and a 1.35 K higher temperature at the heat sink ($T_{\text{sink}} = 294.7$ K at $x=L$) compared to the case without the TD. In this case, the magnetocaloric device would be able to function, because the temperature of TD1 was 0.2 K lower than the heat source and the temperature of TD2 was 0.2 K higher than that of the heat sink.

FIG. 8. Temperature profile shown along the 1D magnetocaloric device without (solid line) and with (dashed line) the TD (the best-case scenario for 20 Hz) after stage II and IV of magnetocaloric thermodynamic cycle.

The impact of the rectification ratio on the temperature difference established between the heat source and heat sink is evident in Fig. 9. The graph presents the temperature of the heat source and the temperature difference in terms of volumetric heat capacity for the operation of the magnetocaloric device at 20 Hz. The rectification ratios $\tilde{R}_{\text{diode1}}$ and $R_{\text{diode1,max}}$ can be read directly from Fig. 7, bottom graph. It is clear that the temperature of the heat source reaches a minimum at around the $\rho c_p = 5 \times 10^4 \text{ J m}^{-3} \text{ K}^{-1}$, which is also the maximum value of $\tilde{R}_{\text{diode1}}$. The temperature difference ΔT is almost constant for $\rho c_p > 4 \times 10^4 \text{ J m}^{-3} \text{ K}^{-1}$, but the temperature shift towards higher temperatures is clear. To design a caloric device with good performance, the temperature of the heat source must be as low as possible.

FIG. 9. Influence of the volumetric heat capacity (directly related to rectification ratio in Fig. 7) on the temperature of the heat source T_{source} in the caloric device and the temperature difference ΔT between the heat source and the heat sink.

V. CONCLUSIONS

A numerical 1D model based on the finite-difference method and Fourier's heat-conduction law was used to analyze various TDs in a magnetocaloric device. As the TD should operate in magnetocaloric devices at higher operating frequencies, i.e., when the time to transfer the heat is very short, the material properties must be tuned to absorb or reject the maximum amount of heat. Therefore, the emphasis of the parametric analysis was on the thermal rectification under transient conditions, which is presented in terms of a time-integrated rectification ratio⁵⁶.

The maximum magnetocaloric rectification ratio R_{diode} under transient conditions was up to 295-times larger than the quasi-steady-state $R_{\text{steady-state}}$, and the time-integrated rectification ratio $\tilde{R}_{\text{diode1}}$ was up to 12-times higher than $\tilde{R}_{\text{steady-state}}$ for the same time interval under quasi-steady-state conditions. Further tests on the TD were performed in a 1D magnetocaloric device for the best-case scenario at 20 Hz, resulting in a 0.2-K temperature difference between the TD and each heat exchanger, meaning a total ΔT equal to 3.4 K.

The results of a simple analysis indicate the great potential of SS TDs in magnetocaloric cooling devices; however, the design and development of TDs will be a challenge for some time. The most promising candidate materials are solid-solid first-order phase-change materials (due to the transient operation, the change of the specific heat capacity also plays an important role), which exhibit a very high and sharp discontinuous change at the critical temperature^{51,52,59} or solid-liquid phase-change materials⁶⁰, which can be tuned to the desired critical temperatures.

Further research should investigate rectification in other caloric refrigeration technologies, because the operating temperatures differ. This analysis could be extended to more complex materials that also exhibit temperature-dependent specific heat capacity and/or density and for different dimensions of TDs.

SUPPLEMENTARY MATERIAL

See supplementary material for the details of the numerical model.

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DATA-AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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