Low-cycle fatigue life of thin-plate auxetic cellular structures made from aluminium alloy 7075-T651

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Abstract

In the article, low-cycle fatigue life durability of thin-plate auxetic cellular structures is compared to the thin-plate specimens of standard shape. Both the re-entrant auxetic cellular structures and the standard specimens were cut from a 2 mm thick stripe of aluminium alloy 7075-T651. First, a fatigue-life curve and a cyclic curve were determined for the standard specimens. A special anti-buckling device was applied to prevent the bending of the specimens. The same experimental arrangement was then applied to determine the low-cycle fatigue life of auxetic structures. In the continuation, the most appropriate method was selected to calculate the fatigue life on the basis of the measured fatigue-life curves. For this purpose, Abaqus and SIMULIA fe-safe software were applied. The best predictions for the standard specimens were obtained with the Brown-Miller method. Finally, the selected method was applied to predict the low-cycle fatigue life of re-entrant auxetic cellular specimens.

Key words

Low-cycle fatigue, Aluminium alloy, Auxetic, Theoretical-experimental approach, Damage models

Nomenclature

b	[-]	Fatigue strength exponent
c	[-]	Fatigue ductility exponent
f_y	[MPa]	Yield strength
f_u	[MPa]	Ultimate strength
l_0	[mm]	Gauge length (extensometer)
n'	[-]	Cyclic strain hardening exponent
E	[MPa]	Young's modulus
K'	[MPa]	Cyclic strain hardening coefficient
K_t	[-]	Surface finish factor
N	[-]	Number of cycles to crack initiation
R	[-]	Stress ratio
ε	[-]	Strain
ε_a	[-]	Strain amplitude

This article should be cited as: Tomažinčič D, Nečemer B, Vesenjāk M, Klemenc J. Low-cycle fatigue life of thin-plate auxetic cellular structures made from aluminium alloy 7075-T651. *Fatigue Fract Eng Mater Struct*. 2019;1–15. <https://doi.org/10.1111/ffe.12966>

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ϵ_f'	[-]	Fatigue ductility coefficient
ϵ_{nom}	[-]	Nominal strain
ϵ_p	[-]	Plastic strain
ϵ_{true}	[-]	True strain
ν	[-]	Poisson's ratio
ν_{AUX}	[-]	Poisson's ratio of the auxetic structure
ν_{RVE}	[-]	Poisson's ratio (Representative Volume Element)
ρ	[g/cm ³]	Density
σ	[MPa]	Stress
σ_f'	[MPa]	Fatigue strength coefficient
σ_m	[MPa]	Mean stress in the cycle
σ_{max}	[MPa]	Maximum stress in the cycle
σ_{nom}	[MPa]	Nominal stress
σ_{true}	[MPa]	True stress
Δl	[mm]	Length change
$\Delta \epsilon$	[-]	Strain range for the cycle
$\Delta \epsilon_n$	[-]	Normal strain
$\Delta \gamma$	[-]	Shear strain

1. Introduction

To reduce weight and improve the carbon footprint of transport systems, aluminium alloys are widely used in the automotive sector for different structural elements. If high-strength, high crash-worthiness or long-life requests are to be met, a suitable material choice are aluminium alloys from the 7xxx family. For this reason, the Al7075-T651 alloy was applied in our research. The fatigue-life data for this alloy is available in the literature (Boller and Seeger¹, Agius et al.², Turkmen et al.³, Meininger et al.⁴). However, it is difficult to find in literature reliable fatigue-life data for thin-sheets and the abrasive water-jet (AWJ) cut specimens. Especially, determination of fatigue-life properties of thin aluminium specimens is a challenge due to their low buckling force. For this reason, an innovative adaptable anti-buckling device was applied in our case that enables fully reversed strain-controlled experiments up to a 2% strain amplitude even for 2 mm thick specimens. That is why we decided to determine the fatigue properties of the applied Al7075-T651 alloy on our own to get true material characteristics for the applied raw material and to improve the fatigue-life predictions of auxetic structures that were studied in this research.

Auxetic structures are a new type of generic structural elements. They are cellular structures with different generic (topological and morphological) geometries, but their common characteristic is that they manifest a negative Poisson's ratio on the macro-structural scale (Yeganeh-Haeri et al.⁵, Lakes⁶, Milton⁷, Shan et al.⁸, Novak, Vesenjask and Ren⁹). This means that the tensile-loaded structure gets

wider in the direction which is perpendicular to the loading direction, and the flexural dimension of the compressive-loaded structures is decreased. Auxetic structures are cellular structures of the future with a tremendous applicability potential in automotive, military and aeronautic industries (Novak et al.¹⁰, Alderson A. and K.L.¹¹, Grima et al.¹², Gaspar et al.¹³, Alderson et al.¹⁴). Up to date, only static and some crash performance of auxetic structures have been tested and numerically evaluated (Körner and Liebold-Ribeiro¹⁵, Taylor et al.¹⁶, Warmuth et al.¹⁷, Novak et al.^{18,19}, Lijun and Weidong²⁰), but information on their fatigue properties is not available in the literature. On the other hand, some investigations of the fatigue behaviour of the lotus-type cellular structures have been published by Kramberger et al.^{21,22}, but these structures are not auxetic. Therefore, the information on the fatigue behaviour of the auxetic cellular structures is needed to find their way into application in a highly dynamic environment, where the loads are significantly changing over time either due to specific operating conditions or vibration produced by the technical system itself.

For this reason, the low-cycle fatigue properties of the planar re-entrant auxetic cellular structure were first determined using the strain-controlled experiments. Then a numerical approach for predicting their fatigue life was developed. One of the research objectives was to apply a commercially available finite element code (Abaqus) and a fatigue-life prediction code (SIMULIA fe-safe) so that the method could be transferred to the industrial environment. Since the critical cross-section of the auxetic structure is significantly inhomogeneous with many large-scale voids, a definition of the critical damage in numerical simulations is especially important. Furthermore, due to the inhomogeneous elastic properties of the auxetic structure all the material links (struts) between the cells in the critical cross-section are often not ruptured at the same time, which complicates the fatigue-life prediction in the process of a progressive propagation of cracked material links. On the other hand, the selected auxetic structure for this research has a well-defined geometry that should behave in a predictable manner during the low-cycle fatigue process.

All the specimens were prepared with an abrasive water-jet cutting process, which means that some roughness is present at the geometry border of the specimens. It is well known from the literature that even small geometrical inhomogeneities at the specimen surface cause micro-notch effects from which initial fatigue cracks develop, thus lowering the fatigue life of the structure (Tomažinčič et al.²³). Usually, this effect is considered by applying correction factors for the surface roughness. In our case, we estimated the surface-roughness correction factor using the reverse-engineering approach. For the standard specimen, we made fatigue life predictions for: i.) ideal geometry without any imperfections and ii.) a real cut boundary of the specimen by modelling it with NURBS curves on the basis of a precise 2D scan of the specimen. Furthermore, we compared the fatigue life

calculations for different methods that are available within the SIMULIA fe-safe software and selected the most appropriate method for our material.

The article is structured as follows. After the introductory section, the performed static and dynamic experiments are described. This is followed by numerical analyses for the standard specimens that were carried out to determine the surface-roughness correction factor and select the most appropriate method for the low-cycle fatigue-life prediction. Then the fatigue lives of the tested re-entrant auxetic cellular structure are calculated and the obtained results are discussed. The article ends with a concluding section, acknowledgement and a list of references.

2. Experiments

2.1. Static tensile tests for standard specimens

First, static tensile experiments were carried out in order to acquire σ - ε characteristics of the Al7075-T651 alloy. On the basis of the σ - ε diagram, loading levels for the fatigue-life experiments were determined. The specimen shape was determined according to the ASTM E606 standard with a modification for the extensometer installation. The same applies for the static tensile and the low-cycle fatigue experiments – see Fig. 3. The specimens were cut with an abrasive water-jet machine from a 2 mm thick sheet metal plate. The smallest cross-section was 24 mm². All the specimens were oriented in the same direction to avoid the influence of material anisotropy. The cut edges and the flat surfaces of the specimens were not additionally machined to avoid necessary definition of the roughness-correction factors for the fatigue-life simulations. Before the specimens were tested, they were scanned with a precision 2D scanner. The scans were imported to the AutoCAD software, where the realistic shape of the 25 mm long section in the middle of the specimen was reconstructed with the NURBS curve – see Fig. 3. 25 mm is also the gauge length l_0 of our extensometer. This model was later applied to numerically assess the influence of the edge roughness on the predicted fatigue life – see section 3.

Two static tensile tests were performed on the MTS 100 kN hydraulic testing machine. A force was measured with a 100 kN load cell and strains were measured with a mechanical extensometer MTS 634.11F-24 and a MTS-certified optical extensometer – see Fig. 1. The output from the mechanical extensometer was also applied to a calibrate contact-less optical extensometer, since it is impossible to attach the mechanical extensometer to the auxetic structures. Data acquisition from the optical extensometer was acquired using the add-on software module in the MTS control software. The static tensile tests were performed according to the ASTM E8/E8M standard with the 0.5 mm/min loading rate at an ambient temperature of 23 °C.

An example of the first specimen rupture can be seen in Fig. 1, the measured engineering σ - ε curves are presented in Fig. 2 and typical material parameters are listed in Table 1, where E is the material Young's modulus, f_y is yield strength and f_u is ultimate strength. It can be concluded from Fig. 2 that the static tensile characteristics have very low scatter.

Fig. 1. Static tensile test of the standard specimen: a- testing machine with data acquisition equipment, b- unloaded state, c- ruptured specimen

Fig. 2. Engineering σ - ε curves for thin sheet Al7075-T651 standard specimens

Fig. 3. Standard specimen for the static tensile and low-cycle fatigue experiments

2.2. Static tensile tests for re-entrant auxetic cellular specimens

The outline of the re-entrant auxetic cellular specimens did not have a standard geometry – see Fig. 4. Since the same specimen shape was applied for the static tensile tests and the low-cycle fatigue experiments, a transition zone between the fixing parts of the specimen at both ends and the true auxetic geometry in the middle was introduced. Its function is a relatively smooth transition between the un-notched part of the specimen, which is attached to the testing machine, and the highly notched part that is typical for the auxetic structures in order to assure the fatigue-crack rupture in the middle of the specimens. The smallest effective cross section of the specimen is 13.2 mm² in the longitudinal direction. The specimens were cut from the sheet-metal plate in the same direction as the standard specimens. Before the experiments were performed, the geometry of the real auxetic specimens was acquired and transformed to the CAD programme in the same manner as for the standard specimens.

Fig. 4. Geometry of the auxetic specimen (above) and ruptured material links after the static tensile test (below)

The static tensile tests for the auxetic specimens were also performed on the 100 kN MTS hydraulic machine, but only the optical extensometer was used in this case. The nominal testing length l_0 was determined using the calibration template – see Fig. 5. The loading rate and ambient temperature were the same as in the case of the standard specimens. In Fig. 5, the tensile test is shown for the initial state, the mid-state with no material links ruptured and the final state with all the material links ruptured. Two static tensile tests were performed for the auxetic specimens. The resulting engineering σ - ε curves are presented in Fig. 6.

Fig. 5. Static tensile test for the auxetic specimen

We can see in Fig. 5 that the static tensile rupture is asymmetrical according to the longitudinal axis of the specimen. All the cracks appeared at the material links oriented approximately at the angle of 45° relative to the first principal stress, which indicates a tough material behaviour. All cellular links

are broken at the same time. Again, the scatter of the engineering σ - ε curves is small, indicating a repeatable static behaviour of the structure. For the re-entrant auxetic cellular specimens the Poisson's ratio ν_{AUX} was also determined from data in Fig. 5:

$$\nu_{AUX} = -\frac{\text{transverse strain}}{\text{longitudinal strain}} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{0.6/39.2}{1.9/34.1} = -0.27, \quad (1)$$

The Poisson's ratio ν for the standard specimens was equal to $\nu_{RVE} = 0.33$.

Fig. 6. Engineering σ - ε curves for the auxetic specimens

2.3. Fatigue-life experiments of the standard specimens

A series of the low-cycle fatigue life experiments was performed for the standard specimens to determine the cyclic and the Coffin-Manson curve²⁴ for the basic material Al7075-T651. All the fatigue-life experiments were carried out with the 100 kN MTS hydraulic testing machine.

The specimens for the low-cycle fatigue experiments had the same shape as the specimens for the static tensile tests – see Fig. 3. The low-cycle fatigue experiments were strain controlled and performed for full reversal dynamic loading ($R_\varepsilon = -1$). The strains were measured with the extensometer MTS 634.11F-24. Because the specimens were loaded up to the strain amplitude $\varepsilon_a = 1.5\%$, there was a danger of buckling in the compressive part of the loading cycle due to the low thickness of the specimens (2 mm thick specimens for the first testing series and 3 mm thick specimens for the second testing series). To prevent the specimen buckling, a special adaptive anti-buckling device was applied to guide the specimens in the compressive regime – see Fig. 9. A 0.1 mm thin strip of the PTFE foil with a declared friction coefficient of 0.04 was installed between the specimen and the buckling device to reduce friction. The remaining friction was measured with a custom-made load cell. The anti-buckling device was attached to the lower jaws of the testing machine.

To reduce the number of specimens needed for the cyclic curve, two types of low-cycle experiments were performed. First, two specimens of each thickness were exposed to the multiple-step test. In this test, the specimen was loaded with 20 loading cycles at each strain amplitude level. It turned out that 20 loading cycles per strain amplitude level were enough for the elastic-plastic hysteresis loops to stabilise. In each consecutive loading block, the strain amplitude was increased by 0.25%. The test was stopped when the specimen ruptured. The results of these experiments were used only to determine the cyclic curve of the Al7075-T651 alloy. The other family of low-cycle fatigue experiments consisted of the strain-controlled constant-amplitude loading at the prescribed amplitude-strain level until the rupture of the specimen. The results of these experiments were used

to determine the cyclic and the Coffin-Manson curve of the Al7075-T651 alloy. The low-cycle fatigue experiments spanned in the range between 0.25% and 1.5% of the strain amplitudes. The loading frequency ranged from 0.5 Hz for the lower strain amplitude levels to 0.2 Hz for the highest loading levels.

The closed hysteresis loops from one of the multiple-step tests are shown in Fig. 7 and all the results of the constant-amplitude experiments are shown in Fig. 8. The material showed a slight cyclic softening only for the first few loading cycles at the highest strain-amplitude levels. The scatter of the hysteresis loops was low.

Fig. 7. Closed hysteresis loops of 2 mm thick standard specimen from the Al7075-T651 alloy

Fig. 8. Coffin-Manson fatigue-life curve for the Al7075-T651 alloy: experimental results and modelled curve

2.4. Fatigue-life curves for the Al7075-T651 alloy

The peaks of the stabilised hysteresis loops represent the points of the cyclic curve. The smallest strain amplitude that was considered for the cyclic curve was $\varepsilon_a = 0.25\%$ and the largest was $\varepsilon_a = 2.0\%$. The σ - ε cyclic curve was modelled with the Ramberg-Osgood model (see Dowling²⁵ or Haibach²⁶ and Fig. 7):

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'} \right)^{\frac{1}{n'}}, \quad (2)$$

where E represents the Young's modulus of the material, n' is the exponent of the cyclic curve and K' represents stress at specific plastic strain ε_p of 1 (100%). Since the low-cycle fatigue data are scattered to some degree, the three parameters were determined with a real-valued genetic algorithm (RVGA) as presented by Klemenc and Fajdiga²⁷. The applied cost function in RVGA was a sum-of squared distances between the measured data points and the analytical model. The resulting parameters for the cyclic curve of our Al7075-T651 alloy are presented in Table 1.

The fatigue life results from fully-reversal constant-amplitude loading in Fig. 8 were modelled using the Coffin-Manson equation in the following form (see Dowling²⁵, Coffin²⁸ or Manson²⁹):

$$\frac{\Delta \varepsilon}{2} = \frac{\sigma_f'}{E} (2N_f)^b + \varepsilon_f' (2N_f)^c, \quad (3)$$

where E is the material Young's modulus, σ_f' is a factor that determines the intercept of the elastic part of the ε - N curve, ε_f' is an intercept of the plastic part of the ε - N curve, and b and c are the exponents of the elastic and plastic parts of the ε - N curve, respectively. The four parameters of the

Coffin-Manson equation were again determined with a variation of RVGA as described by Klemenc and Fajdiga²⁷ due to the scatter of the fatigue-life data. Also, in this case the applied cost function was a sum of squared distances between the measured data points and the analytical model. The parameters of the Coffin-Manson curve for a 50% rupture probability are presented in Table 1. We can see from Table 1 that the Young's modulus obtained from the cyclic curve was a little higher than that from the static tensile test. Due to the phenomenon under consideration, the value of E from the cyclic curve will be applied in the continuation of the article.

Table 1: Material characteristics of the Al7075-T651 alloy

2.5. Low-cycle fatigue experiments for the re-entrant auxetic cellular specimens

The low-cycle fatigue experiments for the four re-entrant auxetic cellular specimens were displacement controlled. For this reason, the output of the LVDT sensor that is built into the 100 kN MTS testing machine was applied. The loading was fully reversal according to the displacement ($R_x = -1$), the testing frequency was 0.2 Hz and the ambient temperature was 23 °C. During each experiment, the force was measured with the 100 kN load cell and the relative displacement of the specimen was measured using the optical extensometer with the help of the two point marks at the side of the specimen. To determine the sequence of material link ruptures during the cyclic loading, one figure from the optical extensometer was saved every second of the experiment. To prevent buckling, the adaptive anti-buckling device was applied, as well as for the standard specimens. The anti-buckling device was mounted in such a manner that all the longitudinal material links were recorded by the camera of the optical extensometer. The friction between the specimen and its guidance was also measured. The experimental arrangement is presented in Fig. 9.

Fig. 9. Low-cycle fatigue test of auxetic specimen: experimental arrangement (above) and acquired figures during the experiment (below)

The loading displacements were determined according to the results of the static tensile tests of auxetic specimens. The four displacement loading levels were $\Delta l = \pm 1$ mm, ± 0.5 mm, ± 0.4 mm and ± 0.35 mm. The loading cycles were repeated until the complete rupture of the specimen. The experimentally determined fatigue lives are presented in Table 3 in section 3 together with the results of the numerical simulations. It can be seen from the comments in Table 3 that in three out of the four cases not all the material links were ruptured at the same time. For the loading level of $\Delta l = \pm 1$ mm, the two middle longitudinal material links were broken first, which was followed by the rupture of the left-most and the right-most longitudinal material link. For the loading level of $\Delta l = \pm 0.5$ mm, first the three right-most material links were broken. The last left-most link was broken outside the critical area. For the loading level of $\Delta l = \pm 0.4$ mm, all the four material links were broken almost at

the same time with the left-most link only with a partial rupture. For the loading level of $\Delta l = \pm 0.35$ mm, all the longitudinal material links were broken in the same loading cycle, but the force dropped constantly from the beginning of the test. We can conclude from these results that often the first ruptures occurred in the middle of the specimen, where the structure was stiffer. In those cases, the last rupture was at the outer most longitudinal links, which survived many more loading cycles than the middle longitudinal links. This, of course, is a consequence of the displacement-control loading and the force after the first ruptured links was dramatically reduced between the complete rupture of the specimen. From the recorded figures during the dynamic tests, we have also observed the auxetic nature of the specimens, which expand in the lateral direction when loaded with the tensile force, and shrink in the lateral direction when loaded with the compressive force.

3. Numerical simulations of the low-cycle fatigue experiments

The fatigue-life simulations were divided into two parts. First, the fatigue-life simulations were performed for the ideal and realistic shape of the standard specimens with the following objectives:

- To assess the influence of the edge roughness to the estimated fatigue life;
- To select the most appropriate method for predicting the fatigue life of the re-entrant auxetic cellular structures that are manufactured from the Al7075-T651 alloy.

Finally, the fatigue lives were numerically predicted for the auxetic structures in order to validate the presented approach for calculating the durability of inhomogeneous structures with significant geometrical voids in the critical cross section. In each case, the stress-strain distribution in the 3D model was determined using the implicit Abaqus code and the fatigue lives were predicted using the SIMULIA fe-safe code.

3.1. Fatigue life prediction for the standard specimens

To estimate the influence of the edge roughness on the fatigue life predictions, two finite element models were built first. Each of the two finite element models represent the narrow middle part of the specimen that is 2 mm thick, 12 mm wide and 25 mm long. One finite element model was built for an ideal case without edge roughness due to the abrasive water-jet cutting process. The other finite element model had a rough cut surface that was determined from the planar surface with rough edges. These edges were modelled as described in section 2.1. To build the finite element model, the top surface was first meshed with quadrilateral planar elements that were further extruded into solid elements using the *Hex Sweep* function in the Abaqus programme. Both finite element models had approximately 28000 C3D8R finite elements and 33000 nodes. The boundary conditions were as follows – see Fig. 10:

- Fixed support at one side of the specimen with a possibility of lateral expansion or shrinkage;
- A pre-defined displacement Δl at the other side of the specimen along its longitudinal axis.

Fig. 10. Boundary conditions for FE simulations of the standard specimen

For each of the two finite element models, five load steps were defined – see also Fig. 11. The first step was tensile loading $+\Delta l$. This was followed by compressive loading $-\Delta l$ (step 2) and unloading state $\Delta l=0$ (step 3 – Fig. 11a). Steps 1 and 2 were repeated as load steps 4 (Fig. 11b) and 5 (Fig. 11c).

Fig. 11. Stress distribution in the specimen with rough edges for the displacement $\Delta l = \pm 0.25$ mm

For the material model, a multi-linear plastic-kinematic model was applied. The alloy density ρ is $2.7...2.81$ g/cm³, [25]. The engineering σ - ε cyclic curve from section 2.3 was transformed into the true-stress/true-strain σ - ε curve using the following equations – see Dowling²⁵:

$$\sigma_{true} = \sigma_{nom}(1 + \varepsilon_{nom}) \quad (4)$$

and

$$\varepsilon_{true} = \ln(1 + \varepsilon_{nom}). \quad (5)$$

Finite element simulations were made for the displacement boundary conditions that are adequate to the four amplitude-strains ε_a : 0.01, 0.005, 0.00233 and 0.00198. These are the amplitude-strain loading levels at last two fatigue life data points. When calculating the amplitude displacement for the respective amplitude strain, the following equation was used – see Table 2:

$$\varepsilon_a = \frac{\Delta l}{l_0}, \text{ i.e.: } \Delta l = 0.01 \cdot 25 = 0.25 \text{ mm}. \quad (6)$$

After the stress-strain distributions had been calculated for the two finite element models with the implicit Abaqus code, the maximum and minimum stress and strain reversal points from the 4th and 5th load step were imported to the SIMULIA fe-safe code. The stress and strain tensors were considered in the centre of each finite element. Since a noticeable roughness due to the sheet-metal rolling exists on the top and the bottom surface of the specimen, it was considered separately via the surface quality factor K_t in the SIMULIA fe-safe code. A value of $K_t = 1.05$ ^[30] was used in our simulations.

To calculate fatigue damage from the stress-strain distribution, the fatigue-life parameters from section 2.3 and 2.4 were considered. The critical plane approach combined three different criteria for calculating the fatigue damage:

- The Brown-Miller model with a Morrow correction for the mean stress that considers normal strains and shear strains in the critical plane (Brown and Miller^{30,31}):

$$\frac{\Delta\gamma}{2} + \frac{\Delta\varepsilon_n}{2} = 1.65 \frac{(\sigma'_f - \sigma_m)}{E} (2N_f)^b + 1.75 \varepsilon'_f (2N_f)^c, \quad (7)$$

- The Coffin-Manson model with a Morrow correction for the mean stress that considers normal strains in the critical plane (Dowling²⁵):

$$\frac{\Delta\varepsilon}{2} = \frac{(\sigma'_f - \sigma_m)}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c, \quad (8)$$

- The Smith-Watson-Topper model for normal strains (Dowling²⁵):

$$\frac{\Delta\varepsilon}{2} \sigma_{\max} = \frac{(\sigma'_f)^2}{E} (2N_f)^{2b} + \sigma'_f \varepsilon'_f (2N_f)^{b+c}. \quad (9)$$

The output of the fatigue-life predictions is a fatigue-life damage that is caused by one repetition of the loading cycle. In the case of loading with constant amplitudes, the number of loading cycles to failure N is calculated as the inverse damage (Stephens et al.³², Liu³³):

$$N = \frac{1}{\text{Damage}}. \quad (10)$$

A comparison of the log-life that was calculated with SIMULIA fe-safe for the two finite element models which correspond to the standard specimens is presented in Fig. 12. The results for all the considered amplitude strains are presented in Table 2.

It can be seen from the presented results that the Brown-Miller and the Coffin-Manson criteria with the Morrow correction for the mean stress outperform the Smith-Watson-Topper criterion. This could be anticipated, because it is known that the latter criterion often yields too conservative fatigue-life estimates. Furthermore, it can be seen from Table 2 that with an exception for the strain amplitude of 1.0% the ideal finite element model results in better predictions than the finite element model with rough edges. This is expected, since the fatigue-life parameters were taken from section 2.3 and 2.4, which means that the edge roughness is already considered by the applied material parameters. Nevertheless, it can be concluded from Table 2 that edge roughness is important, because it can reduce the fatigue life significantly. That is why it is important to consider it adequately.

Fig. 12. Calculated log-life for the Brown-Miller model: $\Delta l = \pm 0.25$ mm (left), $\Delta l = \pm 0.0495$ mm (right); ideal geometry (top), real geometry (bottom)

Table 2: Predicted and measured fatigue life of the standard Al7075-T651 specimens

3.2. Fatigue life prediction for the re-entrant auxetic cellular specimens

The edges of re-entrant auxetic cellular specimens were modelled with NURBS surfaces as described in section 2.2. Then the planar surface was filled within the boundary of the edges and meshed with quadrilateral planar elements. A solid mesh was finally built by a sweep function in the same manner as for the standard specimens. The auxetic specimen was modelled with approximately 139000 C3D8R finite elements and 178000 nodes. The material parameters were kept the same as for the standard specimens. The specimen was clamped at one side and loaded with the prescribed displacements of ± 1 , ± 0.5 , ± 0.4 and ± 0.35 mm (each corresponding to a particular experimental set-up) on the other side. The five load steps were defined in the same way as in section 3.1 and the peak stresses from the last two load steps were considered for calculating the fatigue damage. The same damage criteria were considered as for the standard specimens.

Fig. 13 shows log-lives that were calculated with the Brown-Miller criterion for the four different strain amplitudes. The results are shown in the same scale to make the comparison easier. The limiting number of loading cycles for the white-coloured area is 100. All the fatigue-life predictions for the auxetic specimens are presented in Table 3 with the experimentally determined results.

As evident from the bottom two pictures in Fig. 13, the middle part of the auxetic specimen has a somewhat higher stiffness than the edge, because more damage as a result of higher strains can be observed in the middle of the specimen. Actually, if the material links did not break in the same loading cycle during the experiment, the middle material links were usually the ones that broke first, which is in agreement with the numerical predictions. If we compare the spots with the highest predicted damage to the pictures of broken specimens, we can conclude that the agreement is good in general. For all the loading levels, the most critical parts of the structures are the longitudinal links at the spots with well-expressed curvatures. In these spots, the internal bending moments are the highest and result in the highest stresses and strains. Often (but not always) the fatigue ruptures follow a certain line (see Fig. 13d). However, the fatigue ruptures can also occur randomly, which was the case with the highest loading level (see Fig. 13a).

The predicted load cycles to failure in Table 3 correspond to the fatigue damage equal to one. They are close to the measured fatigue damage for the Brown-Miller and the Coffin-Manson criteria. Again, the Smith-Watson-Topper criterion is too conservative. This result is expected, since the mesh was not too dense and the critical damage of 1 spans over a significant cross section of the material links. Of course, the agreement with the experiment is good only if we compare the numerical results (representing damage initiation) with the first ruptures of the material links.

Fig. 13. Calculated log-life for the Brown-Miller model: a) $\Delta l = \pm 1 \text{ mm}$, b) $\Delta l = \pm 0.5 \text{ mm}$, c) $\Delta l = \pm 0.4 \text{ mm}$, d) $\Delta l = \pm 0.35 \text{ mm}$

Table 3: Predicted and measured fatigue life of the auxetic Al7075-T651 specimens

4. Conclusion

From the experimental and numerical results presented in this paper, the following can be concluded:

- A scatter of the static tensile curves and the low-cycle fatigue results for the basic material Al7075-T651 is low. This has a positive influence for the prediction of the fatigue life in the low-cycle loading regime if the fatigue-life curves are properly modelled.
- When predicting the fatigue lives of the standard and re-entrant auxetic cellular specimens, three different criteria were applied. It turned out that the Brown-Miller and the Coffin-Manson criteria with the Morrow correction for the mean stress outperform the Smith-Watson-Topper criterion, which is too conservative. In general, the best predictions of the fatigue life for the standard specimens were achieved using the Coffin-Manson criterion. On the contrary, the Brown-Miller model was better for the auxetic cellular specimens. This leads to the conclusion that the Brown-Miller criterion should be used when predicting the fatigue life of complex structures made from the Al7075-T651 alloy.
- We also assessed the influence of the edge roughness to the predicted fatigue life. It turns out that the predicted fatigue life can differ up to 20% if the edge roughness is not properly addressed. The smaller the amplitude strain, the bigger the influence of the surface roughness. This means that especially the complex shapes should be carefully machined in order not to introduce the production-process specific geometric inhomogeneities (e.g. too large radii at sharp angle voids) that significantly reduce the fatigue life.
- Static ruptures are more predictable than fatigue ruptures. During the static tensile tests, the specimens broke in a repeated manner. During the low-cycle fatigue experiments, fatigue ruptures appeared also in a random manner. Nevertheless, the ruptured material links in the re-entrant auxetic cellular specimens were also near the critical cross-section with the least material.

Acknowledgements

The research was performed within the framework of the basic research project J2-8186 entitled “Development of multifunctional auxetic cellular structures”, which is financed by the Slovenian Research Agency “ARRS”. Furthermore, authors acknowledge the financial support from the Slovenian Research Agency (research core funding No. P2-0063).

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Table 1: Material characteristics of the Al7075-T651 alloy

Static tensile test			Cyclic curve (R = -1)			Coffin-Manson curve (R = -1) for 50% probability of rupture			
E [MPa]	f_y [MPa]	f_u [MPa]	E [MPa]	K' [MPa]	n'	σ_f' [MPa]	ϵ_f'	b	c
68900	539	596	71500	816	0.0657	114.5	0.0686	-0.0048	-0.3605

Table 2: Predicted and measured fatigue life of the standard Al7075-T651 specimens

ϵ_a [/]	Δl [mm]	Maximum Von Mises stress [MPa]	LOGLife Brown-Miller (Morrow)	N Brown-Miller (Morrow)	N Normal-Strain (Morrow)	N Normal-Strain (S-W-T)	N – experimentally determined
0.01	± 0.25	575 565*	2.229 2.269*	169 185*	198 217*	2 2*	131 151 181 average= 154
0.005	± 0.125	367 354*	3.077 3.129*	1194 1346*	1459 1667*	24 30*	2818 2600 1623 average= 2347
0.00233	± 0.05825	172 165*	4.605 4.696*	40272 49659*	62230 79433*	4385 6053*	145549 1171494 329240 106682 55900 26680 39080 average= 267804
0.00198	± 0.0495	145 140*	5.089 5.206*	122744 160694*	225424 313329*	20893 31623*	47880 714240 98860 average= 286993

* Results for the finite-element model with the ideal shape

Table 3: Predicted and measured fatigue life of the auxetic Al7075-T651 specimens

Δl [mm]	Maximum Von Mises stress [MPa]	LOGLife Brown-Miller (Morrow)	N – Brown-Miller (Morrow)	N – Normal-Strain (Morrow)	N – Normal-Strain (S-W-T)	N – experimentally determined
± 1	603	-4.299	1	1	1	3 (24)*
± 0.5	603	1.103	13	1	1	38
± 0.4	590	1.288	19	137	1	90 (2009)**
± 0.35	575	2.178	151	206	1	240***

* From 3 to 24 load cycles only 5% of the initial force was present for the same displacement amplitude.

** From 90 to 2009 load cycles only 5% of the initial force was present for the same displacement amplitude.

*** From the beginning of the test, the force was reduced to approx. 35% of the initial force at the 240 load cycles, when the final rupture occurred.

Fig. 1. Static tensile test of the standard specimen: a- testing machine with data acquisition equipment, b- unloaded state, c- ruptured specimen

Fig. 2. Engineering σ - ϵ curves for thin sheet Al7075-T651 standard specimens

Fig. 3. Standard specimen for the static tensile and low-cycle fatigue experiments

Fig. 4. Geometry of the auxetic specimen (above) and ruptured material links after the static tensile test (below)

Fig. 5. Static tensile test for the auxetic specimen

Fig. 6. Engineering σ - ϵ curves for the auxetic specimens

Fig. 7. Closed hysteresis loops of 2 mm thick standard specimen from the Al7075-T651 alloy

Fig. 8. Coffin-Manson fatigue-life curve for the Al7075-T651 alloy: experimental results and modelled curve

Fig. 9. Low-cycle fatigue test of auxetic specimen: experimental arrangement (above) and acquired figures during the experiment (below)

Fig. 10. Boundary conditions for FE simulations of the standard specimen

Fig. 11. Stress distribution in the specimen with rough edges for the displacement $\Delta l = \pm 0.25$ mm

Fig. 12. Calculated log-life for the Brown-Miller model: $\Delta l = \pm 0.25$ mm (left), $\Delta l = \pm 0.0495$ mm (right); ideal geometry (top), real geometry (bottom)

Fig. 13. Calculated log-life for the Brown-Miller model: a) $\Delta l = \pm 1$ mm, b) $\Delta l = \pm 0.5$ mm, c) $\Delta l = \pm 0.4$ mm, d) $\Delta l = \pm 0.35$ mm