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Nonlinear phenomena and dynamics of annular laser beam metal droplet generation

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Abstract: Metal droplets are basic elements of many droplet based innovative manufacturing technologies which nowadays are highly demanded in different industrial applications. In the presented laser droplet generation process an annular laser beam pulse is used to melt the wire-end of the vertically fed metal wire. Due to the interplay of surface tension, gravity force, and light-metal interaction, undulating pendant droplet is formed from the molten wire-end. The critical phase of the laser droplet generation process, that essentially influences the process dynamics, is pendant droplet detachment from the solid part of the wire-end. In the paper, different possible detachment scenarios, related nonlinear phenomena, dynamics and process instabilities observed in the forced drop-on-demand and continuous droplet generation process are presented. A nonlinear model of continuous mass spring like resonant detachment regime is described, results of which are in good agreement with the experiments.

Keywords: metal droplets, annular laser beam, droplet detachment, nonlinear dynamics, stability

1. INTRODUCTION

Nowadays many droplet based [1] innovative technologies use uniform-sized metal droplets for different applications, such as joining [2], and 3-D structuring, as well as micro forming, and micro casting. In order to fulfill the requirements of particular applications, various droplet generation systems and processes [3, 4], including laser droplet generation (LDG) from a metal wire, have been developed [2].

In the presented LDG system an annular i.e. ring-shaped laser beam is applied. To generate a droplet, the laser beam pulse is used to melt the end of a fed metal wire and to detach the formed pendant droplet from the solid part of the wire-end [2]. The annular laser beam ensures uniform and symmetrical heating of the wire-end around its circumference. The properties of the annular laser beam are of great importance, especially in the phase of pendant droplet detachment from the solid part of the wire-end, which is the most critical phase of the LDG process.

In the paper nonlinear phenomena and detachment dynamics of drop-on-demand (DoD) and continuous generation of a droplet from a nickel wire are considered. Pendant droplet detachment in DoD generation is influenced by the position of the annular laser beam focus above the pendant droplet neck, resulting into four dynamically different detachment regimes. Among them a reliable droplet detachment is achieved by the Rayleigh-Plateau (R-P) instability [5] based break-up of the molten wire column above the pendant droplet neck. In the continuous droplet sequence generation, it has been observed that pendant droplet detachment is governed by the interaction between the laser pulse frequency and the dynamics of the pendant droplet. Among the inherently unstable detachment regimes, a detachment that relies on laser frequency exited mass spring like resonant oscillations was observed to be the most stable. The corresponding mathematical model has been formulated. By the model the dynamics of the pendant droplet vertical oscillations, time of droplet detachment and corresponding detached droplet diameter can be predicted in dependence on laser pulse frequency.

In the following the annular LDG system and the process of DoD and continuous droplet sequence generation are described, together with the corresponding main process parameters. The results of the characterization of the experimentally observed DoD and continuous generation detachment regimes are then presented and confirmed by theoretical considerations. The mathematical model, results and their agreement with the experiments are presented preceding the conclusions.

2. ANNULAR LASER DROPLET GENERATION SYSTEM AND PROCESS

The scheme of the annular LDG system is shown in Fig. 1(a). The system employs a Nd:Yag pulsed laser of wavelength $\lambda = 1.064$ $\mu$m with a Gaussian beam. To shape the Gaussian beam into an annular beam a pair of axicon has been used. The annular beam is then guided coaxially to the wire axis by a reflective mirror and focused on the wire-end by a convergent lens. The annular laser beam relative intensity distribution $I_{\text{rel}}$ characterised by the IR camera slightly above the focus position is shown in Fig. 1(b). The LDG from the metal wire consists phenomenologically of two consecutive phases: (1) the formation of a pendant droplet, and (2) the detachment of a formed pendant droplet from the solid part of the wire-end. In DoD generation, during the first phase the laser beam is used to melt the wire-end, which is fed into the annular laser beam focus. Under the action of gravity and surface tension, a pendant droplet is formed from the molten wire-end, which is then detached in the second phase of the process by overcoming the surface tension force $F_s$. The main process parameters of DoD generation, shown schematically in Fig. 2(a), are laser pulse $P(t)$ and wire feeding $v_w(t)$ parameters. The first part of the energy $E_w$ of the laser pulse $P(t)$ is used to form a pendant droplet...
from the wire-end. To form a pendant droplet of a larger diameter the wire is fed into the laser beam focus by required feeding velocity \( v_{f}(t) \) during the pendant droplet formation pulse. The wire feeding \( v_{f}(t) \) is also used to assure proper distance \( l_{o} \) of the pendant droplet neck, with respect to the annular laser beam focus, for its detachment. Forced detachment of the formed pendant droplet on demand is achieved by means of a detachment pulse of energy \( E_{d} \).

Fig.1 (a) Scheme of the annular LDG system. (b) Example of the annular laser beam intensity distribution.

A qualitatively different approach to DoD generation is a continuous generation of droplet. In continuous droplet generation the wire is fed continuously with predefined constant velocity \( v_{w} \), and a sequence of periodic laser pulses of energy \( E_{L} \), with frequency \( f_{p} \), is applied as is shown schematically in Fig. 2(b). In this case the pendant droplet can be detached spontaneously due to the action of gravity force \( F_{g} \) alone, or due to the oscillation resonance phenomena of the growing pendant droplet induced by the laser pulse frequency as described in more detail in the following.

### 3. NON-LINEAR PHENOMENA IN DROPLET DETACHMENT

This chapter describes the experiments and corresponding nonlinear phenomena observed in droplet detachment phase of annular laser beam DoD and continuous droplet generation. The LDG process was analyzed based on high speed IR camera images of the laser beam focus region, which was kept within the Argon shielding atmosphere.

#### 3.1 Drop on demand generation

In the experiments of DoD generation, a nickel wire of diameter \( d_{w} = 0.6 \) mm was used. It has been observed experimentally that the pendant droplet detachment phase, beside the detachment pulse parameters is strongly influenced by the distance of the pendant droplet neck position \( l_{o} \) with respect to the annular laser beam focus. To investigate the influence of the distance \( l_{o} \), a set of experiments at different values of \( l_{o} \), starting from 0 to 1.2 mm, with a 0.1 mm step, was performed for different pendant droplet diameters \( d_{p} \) ranging from 0.8 to 1.0 mm, with a diameter step of 0.1 mm. In all the experiments a constant power detachment pulse of energy \( E_{d} = 12.8 \) J and a duration \( t_{d} = 1.6 \) ms was used.

In Fig. 3(a), the IR images show the states of the detached pendant droplet from the wire-end in dependence on the distance \( l_{o} \). The IR images, at the top of each the wire-end is shown, were acquired at time \( t = 2 \) ms after the onset of the detachment pulse. At smaller distances \( l_{o} \), droplet splashes due to droplet explosion are evident. In the region of \( l_{o} \) values from 0.2 to 1.1 mm, successful detachments were achieved. At the largest distance \( l_{o} = 1.2 \) mm the pendant droplet is not detached, and at \( l_{o} = 1.15 \) mm an interesting phenomenon of detached droplet reattachment can be observed as shown in Fig. 4(a). Similar phenomena were observed at other values of the detached droplet diameter \( d_{p} \). To characterise the DoD detachment quantitatively the initial velocity \( v_{d} \) was estimated from the IR images and its dependence on the distance \( l_{o} \) for various droplet diameters \( d_{p} \) is shown in Fig.3(b). It can be seen that, in the case of successfully detached droplets, the velocity \( v_{d} \) decreases with a decrease in the diameter \( d_{p} \), and is within the range from 0 to 0.8 m/s\(^{-1}\). In the case of detached droplet reattachment, the initial velocity \( v_{d} \), is negative, i.e. it is directed upwards, and zero for not detached droplets.

A closer look at the IR image sequence of a successful droplet detachment phase, which is shown in Fig. 4(b), reveals that with the application of an annular laser beam, detachment is caused by Rayleigh-Plateau instability [5] of the liquid column above the droplet neck. In the IR images shown in Fig. 4(b) the time \( t \) is related to the time of application of the detachment pulse. The first image \((t=0.0 \) ms\) shows a formed pendant droplet attached to the
wire-end. In the images acquired at t = 0.15 to 0.6 ms the molten column of wire above the pendant droplet neck, as produced by the annular laser beam detachment pulse, can be seen. As can be clearly seen, in the following images, the molten column undergoes Rayleigh-Plateau (R-P) instability, and at time t \approx 1.2 ms a pendant droplet is detached from the wire-end. The Rayleigh-Plateau instability based droplet detachment results in high repeatability of detached droplet diameter \(d_d\) and low lateral scatter of the detached droplets.

### 3.2 Continuous generation of droplet sequence

In the experiment of continuous generation of a droplet sequence, a nickel wire of diameter \(d_w = 0.25\) mm was used. The wire was fed continuously with a constant feeding velocity \(v_s = 0.06\) m/s. For droplet formation, rectangular laser pulses of power \(P(f_p)\) and duration \(\tau = 0.8\) ms were applied at the selected pulse frequency \(f_p\). Experiments with different values of the laser pulse frequency \(f_p\) within the interval from 50 to 300 Hz, with a 10 Hz step, were performed. The laser pulse frequency dependent laser pulse power \(P(f_p)\) of duration \(\tau = 0.8\) ms was selected so that the average laser power \(P_a = 120\) W was kept constant.

In Fig. 5 the generated droplet diameters \(d_d\), estimated from the IR records of the process, are presented by blue dots and plotted against the laser pulse frequency \(f_p\). It can be seen that, on average, at higher laser pulse frequencies \(f_p\), droplets with a larger diameter \(d_d\) and a lower diameter scatter are generated. Visual inspection and analysis of the IR images of the continuous generation of the droplet sequence has shown that droplet diameter \(d_d\), as well as its variation, are governed by the droplet detachment regime. Depending on the decreasing laser pulse frequency \(f_p\), three different detachment regimes have been observed: (1) spontaneous detachment, caused by gravity alone, (2) resonant detachment, caused by a combination of the effects of gravity and the laser induced pendant droplet oscillation modes, and (3) a Rayleigh-Plateau instability based break-up detachment regime. A more detailed IR image analysis of the resonant detachment regimes at \(f_p = 130, 200\) and 240 Hz showed that the droplet can be detached due to different pendant droplet resonant oscillation modes excited by the laser pulse frequency \(f_p\) during the pendant droplet growth. Analysis of IR records has shown that also to each detached droplet diameter the corresponding detachment frequency \(f_d\) can be associated. The lowest detachment frequency \(f_d \approx 0.25\) Hz corresponds to the largest spontaneously detached droplets, whereas the highest \(f_d \approx 50\) Hz, which equals the corresponding laser pulse frequency \(f_p\), is achieved by detachment due to R-P instability.

From the diagram presented in Fig. 5 we can see that in general at particular laser pulse frequencies \(f_p = [60-130]\) Hz and \(f_p = [170-230]\) several detachment regimes are involved resulting into unstable irregular scatter of detached droplet diameter \(d_d\). The most stable detachment regimes, from the point of view of repeatable droplet diameter and detachment frequency, are the mass-spring and spontaneous dripping regimes at laser pulse frequencies \(f_p\) of 140 and 300 Hz, respectively.

### 3.3 Droplet detachment - theoretical considerations

The above-described experimental observations have been confirmed by theoretical consideration of the droplet detachment regimes. Spontaneous detachment occurs when the droplet gravity force \(F_g\) alone overcomes the maximum surface tension force \(F_t = \pi d_d \gamma\), where \(\gamma\) denotes surface tension. In the case of the used nickel wire of diameter \(d_w = 0.25\) mm, the theoretical condition of spontaneous droplet detachment \(F_g = F_t\) yields a droplet diameter of \(d_d = 3.25\) mm (green solid line in Fig. 5). The result is in a good agreement with the diameter of experimentally generated droplets at laser pulse frequencies above 240 Hz what confirms their spontaneous detachment due to the gravity force.

In order to provide a simplified theoretical explanation for the detachment of droplets within the laser pulse frequency interval \(f_p = [200, 250]\) Hz, we considered Rayleigh normal mode frequencies \(f_{lm}\) of the free surface droplet oscillations of small amplitudes [6] defined by:

\[
f_{lm} = \frac{1}{2\pi} \sqrt{\frac{\pi l(l+1)(l+2)}{\rho d_d^4}}
\]

Here, \(\rho\) denotes droplet density and \(l\) the index of droplet axially symmetric normal modes \((m=0)\). The forms of the second and the third normal modes, \(l = 2\) and 3, are
graphically presented in Fig. 6(a) and 6(b) By plotting the frequencies $f_{s,0}$ and $f_{p,0}$ as functions of the droplet diameter $d_0$ (Eq. (1)) it can be seen in Fig. 5 that the droplet diameters generated at laser pulse frequencies within the intervals $f_{s,0}=[170,250]$ Hz coincide very well with the third (blue dashed line) and second (blue solid line) normal mode frequencies. Based on this, it can be concluded that droplet detachment at these laser pulse frequencies is caused by the gravity and due to the force caused by Rayleigh normal mode like resonant oscillations of the pendant droplet.

Fig.6 (a) The second and (b) the third Rayleigh normal mode form of free surface droplet.

At lower laser pulse frequencies within the interval $f_p=[60, 190]$ Hz the pendant droplet oscillates as a mass suspended by a spring. Simplified, the detachment of droplets of diameter $d_0$ at a corresponding laser pulse frequency $f_p$ can be explained by means of the mass-spring resonant frequency:

$$f_k = \frac{1}{2\pi} \sqrt{\frac{k(d_0)}{m_0(d_0)}}$$ (2)

In Eq. (2), $k$ and $m_0$ denote a spring constant and the pendant droplet mass, respectively, both of which are dependent on time $t$ via the droplet diameter $d_0$. By plotting the droplet diameter $d_0$ as a function of the resonant frequency $f_k$ (Eq. (2)), as shown in Fig. 5, it can be seen that in the case of the laser pulse frequency interval $f_p=[170,250]$ Hz, the droplet diameter $d_0(f_p)$ (shown as a red solid line) coincides very well with the upper branch and $d_0(2f_p)$ (shown as a red dashed line) with the bottom branch of the experimental droplet diameter.

At the lowest laser pulse frequency $f_p = 50$ Hz, the wire length, fed between the successive pulses, is sufficient for droplet detachment due to R-P instability, to take place. The R-P instability based detachment is, in this case, accompanied by a relatively large scatter of the diameter of the detached droplets (Fig. 5).

4. MODELLING

The experimental observation of the mass-spring like oscillation of the pendant droplet within the interval $f_p=[60, 190]$ Hz led to formulation of a theoretical model of droplet dynamics and detachment. In the model, the pendant droplet system with its complex dynamics was reduced to a nonlinear time dependent mass-spring system where the position $z$ of the point mass in the model corresponds to the droplet centroid vertical position and the spring attachment position $z_a$ corresponds to the position of the droplet neck. The low dimensionality of the model corresponds to the result of a nonlinear analysis of LDG time series [7] which determined the embedding dimension not larger than 5.

The model equation was a second order differential equation:

$$m_d(t)\ddot{z}(t) + c(t)\dot{z}(t) + k(t)z(t) + m_d(t)g = k(t)z_a(t).$$ (3)

Here, $g$ denotes the acceleration of gravity. The time-varying droplet mass $m_d(t)$, damping coefficient $c(t)$, spring stiffness $k(t)$, and spring attachment position $z_a(t)$ were determined as follows. The droplet mass time dependence $m_d(t)$ is shown in Fig. 7(a). Starting with the initial mass $m_0$, droplet mass $m_d$ was linearly increased by $\Delta m = \rho \pi T d_0^2/4$ during each laser pulse due to adding of the melted portion of the wire. Here, $\rho$ is density and $T = 1/f_p$ is period of the laser pulses. The damping coefficient $c(t)$ time dependence was estimated by [8]:

$$c(t) = \mu (3m_d(t)/4\pi T)^{1/3}$$ (4)

where $\mu$ is dynamic viscosity. To reproduce the observed droplet dynamics, the damping coefficient had to be multiplied by a factor of 1300 which may be due to rotational flow inside the droplet [9]. The spring stiffness was determined by time-frequency analysis of the experimental $z(t)$ time series which showed resonances at multiples of laser frequency $f_p$. The effective spring stiffness $k$ was calculated at each resonance by the spring-mass normal frequency equation: $k = 4\pi^2 f_p^2 m_0$. The dependence of $k$ on the droplet mass $m_d$ could be described by $k(m) = k_0 (m/m_0)^{0.26}$, where $k_0 = 0.186$ N/m and $m_1 = 1$ kg. The spring stiffness time dependence $k(t)$ was then expressed by $k(t) = k_0(m_d(t))$.

The time-varying spring attachment position $z_a(t)$ was presumed to consist of linear sections as presented in Fig. 7(b). Between the laser pulses, $z_a$ descends with the velocity of feeding $v_o$. During the laser pulse $z_a$ rises to reach the initial value. The amplitude $\Delta z_a$ of this oscillation is $\Delta z_a = v_o(T-\tau)$. Experimentally it was observed that this amplitude value gradually reduced in $t_1 = 0.28$ s to half of its initial value due to melting of the wire between the laser pulses by the large and hot droplet. The piecewise linear $z_a(t)$ was finally smoothed by a 0.13 $T$ running average filter to avoid unrealistic discontinuities in $\dot{z}_a(t)$.

Fig.7 (a) The droplet mass $m$ time dependence. (b) The spring attachment position $z_a$ time dependence.

The model was solved for the laser pulse frequency $f_p$ values between 50 Hz and 200 Hz and for a set of 50 equally spaced initial mass $m_0$ values between
0.04·10⁻⁷ kg and 2.4·10⁻⁷ kg. The detachment conditions were 1) \( m\dot{z} + mg > F_v \), and 2) \( \dot{z} \leq \nu_{det} \), where values \( \nu_{det} = 0 \) and \( \infty \) were used to prevent reattachment and to allow detachment due to the first condition only, respectively.

An example of the modelled (red line) and the corresponding experimental (blue line) \( z(t) \) time series at \( f_p = 100 \) Hz is presented in Fig. 8. The model successfully reproduced the vertical oscillation of a pendant droplet except between \( t = 0.10 \) and 0.15 s, where the pendant droplet in experiment oscillated laterally. The modelled time of droplet detachment denoted by a circle (at \( \nu_{det} = 0 \)) corresponds well with the experimental time of detachment denoted by a square due to the resonance at \( f_p \). The modelled time of detachment at \( \nu_{det} = \infty \) is denoted by a triangle.

![Graph of z(t) time series](image1.png)

**Fig.8** Example of modelled \( z(t) \) time series (red line) at \( f_p = 100 \) Hz compared to the experimental \( z(t) \) (blue line).

The modelled detached droplet diameter \( d_d \) vs. \( f_p \) is presented in Fig. 9 by circles for \( \nu_{det} = 0 \) and triangles for \( \nu_{det} = \infty \) and compared to experimental values which are denoted by blue points. The upper experimental branch consists of droplets which detached due to resonance at the laser pulse frequency \( f_p \). At \( f_p \leq 120 \) Hz also smaller droplets detached due to resonance at \( 2f_p \), which populate the lower branch. Both experimental branches are reproduced very well by the model.

![Graph of detached droplet diameter](image2.png)

**Fig.9** Experimental (blue) and modelled (red) detached droplet diameter \( d_d \) vs. the laser frequency \( f_p \).

### 5. CONCLUSION

In the paper the annular laser beam droplet generation process and corresponding nonlinear phenomena and detachment dynamics in drop-on-demand (DoD) and continuous generation of a droplets from a nickel wire were considered. The results show that annular laser beam droplet generation from a metal wire is very complex and inherently unstable process.

In DoD generation beside the successful pendant droplet detachment, droplet explosion, non-detachment and droplet reattachment have been observed in dependence on the position of the annular laser beam focus above the pendant droplet neck. Reliable forced detachment of pendant droplets can be achieved by the Rayleigh-Plateau instability based break-up of the molten wire column above the pendant droplet neck.

In the continuous droplet sequence generation, it has been shown that pendant droplet detachment is governed by the pendant droplet resonant dynamics excited by the laser pulse frequency. Among the observed inherently unstable detachment regimes, the detachment that relays on exited mass spring like resonant oscillations was finding to be the most stable. The corresponding mathematical model of continuous LDG has been formulated. By the model the dynamics of the pendant droplet vertical oscillations, time of droplet detachment and its diameter can be predicted in dependence on laser pulse frequency. What is more, the model reproduced the region of LDG process stability above the laser pulse frequency of 120 Hz and a simultaneous occurrence of two detached droplet diameter values at laser pulse frequencies below 120 Hz.

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