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Nonlinear Analysis of Mineral Wool Fiberization Process

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Abstract

In this paper, mineral wool fiberization process on a spinner wheel was studied by means of the nonlinear time series analysis. Melt film velocity time series was calculated using computer-aided visualization of the process images recorded with a high speed camera. The time series were used to reconstruct the state space of the process and were tested for stationarity, determinism, chaos and recurrent properties. Mineral wool fiberization was determined to be a low-dimensional and nonstationary process. The 0-1 chaos test results suggest that the process is chaotic while the determinism test indicates weak determinism.

Keywords: Mineral wool, Melt fiberization, Nonlinear analysis, Time series, Visualization method

1 Introduction

In production of mineral wool, melt fiberization on spinner wheels is one of the most critical phases determining the quality of the end product. Knowing the fiberization mechanism well is necessary to make improvements in quality and control of the manufacturing process. Due

to the process complexity and hazardous operating conditions however, there has been little research on the fiberization dynamics, especially on the smaller length and time scales. Consequently, the understanding of fiber formation mechanism is mostly limited to the relatively simple theoretical models of the underlying hydrodynamic instabilities such as the one by Rayleigh [1] and Eisenklam [2] in addition to the empirical models of liquid ligament formation (Hinze and Milborn [3], Kamyia [4], Liu et al. [5]). While these models may partially explain the initial formation mechanism, the dynamics of later fiberization phases, including solidification and breaking of the fibers, remain largely unknown. Westerlund and Hoikka [6] studied the dynamics of the cooling mineral wool fibers numerically but were unable to determine the cause of fiber breakage. On the other hand, a number of macro scale analyses exist studying the relation between the spinner input parameters and the resulting fiber properties (i.e. thickness, length and mechanical properties) while not focusing on the formation mechanism itself. For example, Širok et al. [7, 8] developed a phenomenological model for the fiber thickness on an industrial spinner machine as a function of multiple input parameters.

While the macro scale phenomenological studies have proven partly successful in improving production process control, future improvements require real time, micro scale approach methods, both experimental and numerical. Recently, we have investigated the fiberization process on an industrial spinner [9] using a computer-aided visualization method where images of the process were acquired by means of a high speed camera. The analysis provided some valuable insight in the fiberization process dynamics, but also faced the limitations of

the linear time series analysis. For this reason, we decided to expand the analysis of the time series to the nonlinear methods as the observed process is suspected to be chaotic and exhibit nonlinear dynamics. Nonlinear time series analysis methods have so far been used in a wide variety of different applications ranging from electronics, weather forecasting, traffic systems, medicine and mechanical engineering [10] and may successfully characterize the studied phenomenon even when the linear methods fail. Our main purpose for applying nonlinear methods to the mineral wool fiberization analysis is to characterize the process in terms of chaotic properties, stationarity, determinism and dimensionality. A chaotic process that is deterministic and low-dimensional, may be modeled mathematically with a proper set of equations.

This paper is organized as follows. Section 2 introduces theoretical background of the mineral wool fiberization mechanism. In section 3, experimental setup for melt film visualization is presented along with a brief description of velocity calculation method. In section 4, the obtained melt film velocity time series is analyzed using nonlinear methods and the calculation results are presented. In the final section, results are summarized and our findings are presented.

2 Mechanism of mineral wool fiberization

When a stream of melt falls onto the mantle surface of a spinner wheel (Fig. 1), it is drawn in motion by viscous and adhesive forces, forming a thin film slightly wider than the melt stream. According to Westerlund and Hoikka [6], liquid ligaments form from the melt film due to

hydrodynamic instabilities and then solidify into fibers. Initial melt film disturbances required for development of unstable waves are most likely caused by the Kelvin-Helmholtz instability (Helmholtz 1868, Kelvin 1871) induced by velocity slip between the film and the surrounding air as well as by fluctuations in the melt flow rate. The main wave formation mechanism however is the Rayleigh-Taylor instability (Rayleigh 1900, Taylor 1950), for which a detailed mathematical formulation was provided by Taylor [1]. It occurs because the melt film (i.e., the denser fluid) is pushed towards the surrounding air (i.e., the lighter fluid) by the centrifugal force, causing unstable waves to form on the film surface.

According to Eisenklam [2], such unstable waves grow at different rates depending on their wavelength. Waves below a certain cutoff wavelength (λ_{min}) are fully damped by viscous and surface tension forces while at higher wavelengths where centrifugal force is greater than the aforementioned damping forces, the waves grow exponentially and at different growth rates. It can be assumed that the fastest growing wave (wavelength λ_m) becomes predominant [2] and transforms into the circumferential spacing (s) between emerging liquid ligaments (Fig. 1). A simplified presentation of initial fiber formation mechanism is shown in Fig. 1 where f_0 denotes the spinner wheel rotational speed in [Hz], R the wheel radius and h the liquid film radial thickness. Melt properties are defined by its density (ρ), surface tension (σ), viscosity (μ) and mass flow ($\dot{m}$).

{Figure 1}

According to Eisenklam [2], Rayleigh-Taylor instability occurs for a disturbance growth rate factor $\xi^2 > 0$ - Eq. (1):

$$\xi^2 = \left[(2\pi f_0)^2 R - \frac{k^2 \gamma}{\rho} \right] \cdot k \cdot \tanh(kh) \quad ; \quad k = \frac{2\pi}{\lambda} \quad (1)$$

The wavelength above which the centrifugal force exceeds the surface tension force and the instabilities are no longer fully suppressed is given by Eq. (2):

$$\lambda_{\min} = 2\pi \left(\frac{\rho}{\gamma} \omega^2 R \right)^{-1/2} \quad (2)$$

For $h \geq \lambda_m / \pi$, the fastest growing perturbation with a wavelength of λ_m can be estimated by Eq. (3) [2]:

$$\lambda_m = \sqrt{\frac{3\sigma}{\rho R f_0^2}} \quad (3)$$

Equations (1), (2) and (3) do not take melt viscosity in consideration. Viscosity generally dampens the growth of unstable waves, effectively increasing λ_m [2] and consequently, the ligament spacing (s). Hinze and Milborn [3] experimentally obtained proportionality $s \propto \mu^{1/3}$, suggesting that ten-fold increase in melt viscosity results in approximately 100% increase in ligament spacing (or equally, 50% reduction in number of ligaments attached to the melt film). However, theoretical and experimental ligament formation models by various authors given in [2, 3, 4, 5] agree that the most significant operating parameter is the wheel rotational speed (f_0). Typically, the combined rotational speed and surface tension effect is characterized by

Weber number ($We = \rho(2\pi f_0)^2 R^3 / \sigma$). Eisenklam's formulation for inviscid liquids from Eq. (3) can be rewritten to proportionality $s \propto We^{-0.5}$. For viscous liquids, slightly lower Weber number exponents (in terms of magnitude) were obtained: -0.42 by Kamiya [4] and -0.44 by Liu et al. [5].

It is important to note that formulations cited from [2, 3, 4, 5] were all derived for cold Newtonian liquids while, to our knowledge, no similar models exist for melts. Nevertheless, these formulations are also applicable to silicate melts in the initial phase of ligament formation when melt has a high enough temperature (1200-1300°C) to be treated as a Newtonian fluid [8]. As the melt ligaments cool down, the fluid becomes non-Newtonian and gradually, solidification occurs. Of course, melt fluid dynamics in an industrial spinning machine is much more complex than in case of cold experiments conducted in a laboratory environment. Due to unsteady melt string flow onto the spinner wheel and wheel vibrations, other significant melt film oscillations with wavelengths much larger than λ_m are expected.

Certainly, in the phase of initial ligament formation from the film, wheel rotational speed and velocity slip between the melt film and the wheel surface both have a very significant effect on the ligament spacing as they alter the film Weber number. Larger ligament spacing results in a proportionally lower number of ligaments and consequently, larger ligament diameter, affecting the quality of produced mineral wool. For this reason, we decided to measure melt film circumferential velocity and analyze the acquired time series.

3 Experimental setup

Experiment was conducted on a four-wheel spinner (Fig. 2) that was a part of an industrial mineral wool production line. The region of interest was narrowed to the first spinner wheel where three distinctive regions were present, marked as (A), (B) and (C) on the Fig. 2.

{Figure 2}

Region (A) represents the flow of the melt string onto the wheel to the impingement point. Region (B) is in an area on the wheel surface where a thin melt film is formed. On the film surface, initial fiberization occurs by means of liquid ligament formation. Region (C) denotes the area immediately above the wheel where partly formed fibers move towards the blow-in flow. In this paper, region B was studied using a visualization method. The experimental setup is shown in Fig. 3.

{Figure 3}

Melt flow was visualized using a high speed camera *Fastec Hispec 4 mono 2G* attached to a tripod. 50mm f1:1.2 lenses were used with the aperture fully opened. Camera was placed at a distance of 4.5m from the first spinner wheel ($R = 0.14\text{m}$) and at an angle of 45° relative to the wheel's rotational axis. Camera shutter speed was set to $6\mu\text{s}$ and to a recording rate of 18660 frames per second. Due to the narrow available range of process parameters (such as the spinner wheel rotational speed) dictated by the industrial production process, any parameter variation within this range would lead to roughly the same operational regime. Therefore, a single measurement was made at the spinner wheel rotational speed $f_0 = 113\text{Hz}$.

Melt density, viscosity and surface tension were 2620 kg/m^3 , $570\text{Pa}\cdot\text{s}$ and 0.42N/m , respectively. A series of 2500 images sized 128×328 pixels and with a resolution of 0.37mm was recorded on a nearly vertical segment of the melt film.

In the acquired images, we performed an analysis of the melt film velocity in a window shown in Fig. 4. Due to the melt film transversal velocity (v_x) component being very small compared to the circumferential component (v_y), we selected v_y as the property representative of the investigated process and relevant for both linear and nonlinear time series analysis. The magnitude of v_y directly affects melt film Weber number and consequently, the mechanism of fiber growth. Also, the fluctuations of v_y magnitude are significantly large [9] and result from velocity slip between melt film and spinner wheel surface. The slip most likely occurs due to the effect of aerodynamic drag acting upon the melt film and forming fibers. Velocity slip also exhibits temporal and spatial fluctuations that may arise from the eccentricity (varying thickness) of the melt film [9], hydrodynamic instabilities and tearing of the ligaments. Therefore, the v_y time series contains valuable information about the fiberization process dynamics.

{Figure 4}

To generate the time series required for analysis, we first calculated a velocity field of $\mathbf{v} = (v_x, v_y)$ within the window of interest for every image in sequence. A computer-aided visualization method developed by Bajcar et al. [11] and implemented in *ADMflow* software

(<http://admflow.net>) was used for velocity calculation. The calculation method is based on a simplified form of advection diffusion equation (Eq. (4)) and the calculation of derivatives on a fixed-coordinate grid.

$$\frac{\partial A}{\partial t} + \frac{\partial A}{\partial x} v_x + \frac{\partial A}{\partial y} v_y = D \left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} \right). \quad (4)$$

In Eq. (4), A is the image brightness (grey level) which we assumed to be proportional to the melt radial thickness in the images. x and y are the absolute coordinates, v_x and v_y flow velocities that are of interest and D the diffusion coefficient. For a silicate melt, diffusion coefficient is very low (advection is predominant mass transport mechanism) and can be estimated to $D \approx 10^{-10} \text{m}^2/\text{s}$ (at 1200°C) [12]. Note that in this form with the right side set to 0, Eq. (4) is identical to the one of the optical flow [13].

Derivatives are discretized using central difference method (Equations (5) and (6)):

$$\frac{\partial f}{\partial x_i}(x) \approx \frac{f(x + \Delta x_i) - f(x - \Delta x_i)}{2\Delta x_i}, \quad (5)$$

$$\frac{\partial^2 f}{\partial x_i^2}(x) \approx \frac{f(x + \Delta x_i) - 2f(x) + f(x - \Delta x_i)}{\Delta x_i^2}, \quad (6)$$

Following a discretization scheme (Eq. (7)), differential equations are transformed into linear equations:

$$\begin{aligned}
& \frac{A(i, j, t + \Delta t) - A(i, j, t - \Delta t)}{\frac{2\Delta t}{f_s}} + \frac{A(i + \Delta xy, j, t) - A(i - \Delta xy, j, t)}{2\Delta xy \cdot s} v_x(t, i, j) + \\
& + \frac{A(i, j + \Delta xy, t) - A(i, j - \Delta xy, t)}{2\Delta xy \cdot s} v_y(t, i, j) = \\
& = D \left(\frac{A(t, i + \Delta xy, j) - 2A(t, i, j) + A(t, i - \Delta xy, j)}{(\Delta xy \cdot s)^2} + \frac{A(t, i, j + \Delta xy) - 2A(t, i, j) + A(t, i, j - \Delta xy)}{(\Delta xy \cdot s)^2} \right) \quad (7)
\end{aligned}$$

$v_x(t, i, j)$ and $v_y(t, i, j)$ denote the components of the velocities at the pixel (i, j) of the fluid at time t . Time t is discrete: $t = 0, 1, \dots, R - 1$, where R is the number of images. The integers Δt and Δxy represent the time and space displacements in the number of images in the series and the number of pixels in an image, respectively. f_s is the number of frames per second and s the pixel size in meters.

The number of unknowns exceeds the number of equations as our approach is basically by solving Eq. (4) without the boundary conditions. To increase the number of equations in the model, two methods are used – temporal smoothing and spatial smoothing (velocity gradient penalization). Solution of the system is then obtained using the least squares method.

Once velocity fields of $\mathbf{v} = (v_x, v_y)$ were calculated within the window of interest, we calculated a spatial average of v_y for every image in sequence and formed discrete one-dimensional time series with 2470 points and a sampling rate of 18660Hz (equal to the image acquisition rate).

4 Time series analysis

The v_y time series obtained by the algorithm described in the previous section was first filtered using a low pass Butterworth filter at -40dB/decade with a cutoff frequency of 2000Hz. In Fig. 5, the filtered and unfiltered signal is shown. The magnitude of v_y oscillates between approximately 85m/s and 105m/s.

{Figure 5}

It can be seen that the DC component of the signal (time series) is relatively large compared to the AC component, but does not change over time (for the filtered signal, it is equal to 92.1m/s). Constant value of the DC component suggests that only the AC component is significant for the analysis of fiberization process dynamics. For this reason, we only used the AC component (i.e., the fluctuating part) of the signal in subsequent linear and nonlinear time series analyses of the process. The AC component was obtained by subtracting mean signal value from the filtered v_y signal.

In the next step, power spectrum (Fig. 6.) was generated from the filtered time series using a Fast Fourier Transform (FFT) algorithm.

{Figure 6}

In the spectrum, a first significant peak occurs around 120Hz which is close to wheel rotational speed ($f_0 = 113\text{Hz}$), and also the second and third harmonic frequency peaks are clearly visible at about 240Hz and 360Hz, respectively. Most likely, due to low-frequency pulsations in melt

string and slightly inhomogeneous melt structure, melt film on the wheel is eccentric, with its thickness within our region of interest oscillating as it rotates with the wheel. Thicker areas of melt film are hotter, meaning that viscosity and surface tension is reduced, leading to poorer film adhesion to the wheel and consequently, to an increase in velocity slip and reduction of the film circumferential velocity (v_y) [9]. Therefore, melt thickness oscillations at f_0 and its higher harmonics are also reflected in fluctuations of v_y at approximately the same frequencies. These oscillations are deterministic.

Peaks that occur in the intermediate range between 500-1000Hz may have been caused by wheel vibrations, but it not possible to confirm the cause of oscillations as vibrations were not measured in our experiment. In the frequency range above 1000Hz, we can expect oscillations that are superimposed on the lower frequency deterministic oscillations and are caused by phenomena such as the Rayleigh-Taylor hydrodynamic instability and tearing of the fibers from the melt film. However, a linear analysis of these oscillations is beyond the scope of this article as it was already performed in [9]. Also, v_y time series does not allow for analysis of high frequency phenomena (much above 1000Hz) as the v_y amplitude quickly drops towards zero due to the time series averaging effect induced by velocity calculation algorithm and the low-pass filter at 2000Hz. For this reason, part of the spectrum above 1600Hz is omitted.

While the visual inspection of the time series and power spectrum gives some information about the fiberization dynamics, we have at this point already exhausted the relevant linear

analysis tools and will continue with the presentation of nonlinear analysis results. The basic principle in nonlinear time series analysis is to reconstruct the phase space of the original system from a measured time series of a property which is characteristic for the process (in our case, melt film y velocity). Reconstruction is most commonly performed using the embedding theorem [14, 15] where phase space is reconstructed by means of delay vectors (Eq. (8)).

$$\mathbf{s}_n = (s_n, s_{n+\tau}, s_{n+2\tau}, \dots, s_{n+(m-1)\tau}) \quad (8)$$

The time delay τ is obtained either as the first zero of the autocorrelation function ($R_{xx}(\tau)$) [10] or the first local minimum of the mutual information ($I(\tau)$) [16, 17]. In Fig. 7, mutual information is shown with a solid line and we get $\tau = 19$ as the time delay at the first local minimum of I while the autocorrelation function (dashed line) has its first zero at $\tau = 23$.

{Figure 7}

Based on the inspection of both $R_{xx}(\tau)$ and $I(\tau)$ graphs, we decide to take $\tau = 19$ for our further calculations.

4.1 Dimensionality of the process

After selecting a proper time delay, we also need to determine the minimal embedding dimension m for a proper attractor reconstruction where the reconstruction attractor is equivalent to the original attractor in terms of the system's dynamical properties. For this purpose, false nearest method was used [18].

Minimal required embedding dimension m is determined as the one where the fraction of the false nearest neighbors (fnn) drops to zero, meaning that the reconstructed phase space is correctly unfolded. Points that are close in the reconstructed m -dimensional embedding space have to stay sufficiently close in embedding spaces with $m+1$ and more dimensions. If two points in the phase space are apparently close but do not fulfill this criterion, they are considered false nearest neighbors.

In Fig. 8, a graph of false nearest neighbor fraction (fnn) is given as a function of embedding dimension m .

{Figure 8}

From Fig. 9, we note that the time series low pass filtering reduces minimal embedding dimension by one and we get $m = 4$, indicating that fiberization process is low-dimensional.

For all subsequent calculations, values $\tau = 19$ and $m = 4$ will be used.

4.2 Stationarity test

Stationarity of the time series was tested using the cross prediction error method proposed by Schreiber [19]. With this method, the original time series is divided in r segments that have to be sufficiently long. For i -th segment in the time series ($i = 1 \dots r$) the value of the j -th segment is predicted using a simple linear prediction – Eq. (9).

$$\hat{s}_{N+\Delta n} = \frac{1}{|U_{\varepsilon}(\underline{s}_N)|} \sum_{\underline{s}_n \in U_{\varepsilon}(\underline{s}_N)} s_{n+\Delta n} \quad (9)$$

$\hat{s}_{N+\Delta n}$ is the measurement forecast at the time $N + \Delta n$ and $\underline{s}_N$ is the point in the time series at time N from which the value of $\hat{s}_{N+\Delta n}$ is predicted. $U_{\varepsilon}(\underline{s}_N)$ is the neighborhood size of the point $\underline{s}_N$ with the radius ε and $|U_{\varepsilon}(\underline{s}_N)|$ is the number of points within the neighborhood. We can now define a relative cross prediction error (Eq. (10)) as

$$e = \frac{1}{\langle s_{n+\Delta n} \rangle} \sqrt{\langle (s_{n+\Delta n} - \hat{s}_{N+\Delta n})^2 \rangle} = \frac{1}{\langle v_{y,j} \rangle} \sqrt{\langle (v_{y,j} - \hat{v}_{y,j})^2 \rangle} \quad (10)$$

In our calculation, the time series of v_y was divided into 12 segments 205 points in length. Embedding time $\tau = 19$ and embedding dimension $m = 4$ were used. Cross prediction errors are given in Fig. 9.

{Figure 9}

Cross prediction errors are mostly in the range between 0.4 and 0.8 and have a mean value of 0.69 while the error standard deviation is 0.11. Error mean value and range are relatively high while the standard deviation is several times lower. This indicates high cross prediction error values are partly a consequence of a relatively short time series. It is likely that the time series as a whole is nonstationary but it may contain strong stationary components.

4.3 Determinism test

In addition to the stationarity test, we also tested our time series for determinism using an algorithm proposed by Kaplan in Glass [20]. The determinism test is conducted by first dividing the m -dimensional embedding space to M^m equally sized boxes (a value of $M = 13$ was chosen). For each pass p of the trajectory through the k -th box, a unit vector $\mathbf{e}_p$ is calculated as the norm difference between the trajectory exit and entrance point. An average directional vector $\mathbf{V}_k$ for k -th box is calculated by Eq. (11):

$$\mathbf{V}_k = \frac{1}{n} \sum_{i=1}^n \mathbf{e}_p \quad (11)$$

The idea is that for a deterministic system, the trajectories close to each other should point in roughly the same direction, thus assuring uniqueness of solutions in the phase space which is the basic principle of determinism [21]. If $0 \leq |\mathbf{V}_k| \leq 1$ is the length of the vector $\mathbf{V}_k$, then for a deterministic system we expect $|\mathbf{V}_k| \approx 1$. The determinism factor κ is then calculated as an average $\mathbf{V}_k$ length for all boxes in the embedding space containing at least a minimum required number of trajectory passes (n_{min}) to be significant. In our calculation, we chose $n_{min} = 3$. $K = 1$ denotes an ideally deterministic system while $\kappa = 0$ means a completely stochastic system. In order for the system to be classified as deterministic, κ is required to be reasonably close to 1 (say, $\kappa > 0.7$). For our system we obtained $\kappa = 0.66$ which points to a weak determinism. The

reconstructed embedding space and the pertaining directional vector fields are shown in Fig. 10 as a visual verification of determinism test.

{Figure 10}

4.4 Recurrence plots

To visually inspect the properties of the time series, recurrence plots can be used provided that the system is deterministic [11, 22]. Recurrence plots are particularly suited for detection of aperiodic behavior where the repetitions of the system states follow no simple rule. Recurrence is an approximate repetition of a system state in a reconstructed phase space when, at time i , a trajectory returns into the ε -neighborhood of a point in the phase space where it already was at time j – Eq. (12).

$$M_{ij} = \Theta(\varepsilon - \|s_i - s_j\|) \quad (12)$$

In Eq. (12), M_{ij} can take two values, either $M_{ij} = 1$, which indicates recurrence (a dot is drawn in the recurrence plot at (i,j)), or $M_{ij} = 0$, which means no recurrence occurred. $\Theta()$ is the Heaviside step function that has a value of 1 when $\varepsilon - \|s_i - s_j\| \geq 0$ and 0 elsewhere. In Fig. 11, a recurrence plot is shown for our system. ε value was adjusted to result in the plot recurrence rate of 1%. Recurrence rate is defined as a fraction of points with $M_{ij} = 1$ among all points in the recurrence plot.

{Figure 11}

From Fig. 11, we conclude that our time series cannot be considered stationary as there are clearly visible vertical belts around $i = 700, 1000, 1500$ and 2000 where recurrence rate is significantly lower than in other areas of the plot.

4.5 Recurrence quantification analysis

For a more detailed study of our system's recurrent properties, we employed recurrence quantification analysis [23, 24]. The time series of v_y was divided into 6 sections and the following quantification measures were chosen:

- Determinism (DET) which denotes the ratio between the number of recurrent points that form diagonal lines in the recurrence plot, and all recurrent points (Eq. (12)).

- Laminarity (LAM) which is defined as a ratio between the number of recurrent points that form vertical lines in the recurrence plot, and all recurrent points.

- Average length of diagonal lines in the recurrence plot (L); this quantity is a measure of the average time that two segments of the trajectory are close to each other, lower L value indicating a faster divergence.

- Divergence (DIV) is an inverse value of the largest diagonal line length, $DIV = 1/L_{max}$. Larger divergence means that the trajectories diverge faster and the process is more chaotic.

- Diagonal (L_{entr}) and vertical (V_{entr}) line entropy that reflect the complexity of recurrence plots with respect to diagonal and vertical lines, respectively.

The chosen RQA measures for our system are shown in Fig. 12. Determinism and laminarity do not change significantly between the segments of the time series as the value of both is confined to a range of 0.8-1. Recurrence rate is roughly 0.01 for all segments, meaning that the time series segment length was chosen appropriately [25]. On the other hand, DIV and L measures change more significantly, again indicating the time series may be nonstationary. From the L_{entr} and V_{entr} graphs, we note that the L_{entr} values are higher than V_{entr} , indicating a greater complexity of the recurrence plot in the vertical direction.

{Figure 12}

4.6 0-1 test

We tested our time series for chaos by applying the 0-1 test developed and recently improved by Gottwald and Melbourne [26, 27]. For a scalar time series $x_i(t) = x_1, x_2, \dots, x_N$, the indices $n = 1, \dots, N$ and the constant $c \in [0, \pi]$ we define the translation variables $p_c(n)$ in $q_c(n)$ (Equations (13) and (14)):

$$p_c(n) = \sum_{i=1}^n x_i \cos(ic) \quad (13)$$

$$q_c(n) = \sum_{i=1}^n x_i \sin(ic) \quad (14)$$

Mean square displacement between $p_c(n)$ and $q_c(n)$ is given by Eq. (15):

$$M_c(n) = \lim_{n \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N [p_c(i+n) - p_c(i)]^2 - [q_c(i+n) - q_c(i)]^2 \quad (15)$$

To improve the convergence properties, $M_c(n)$ is modified into $D_c(n)$ – Eq. (16):

$$D_c(n) = M_c(n) - \left(\lim_{n \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N x_i \right) \frac{1 - \cos(nc)}{1 - \cos(c)} \quad (16)$$

Due to the finite length of the time series, limit sign (*lim*) can be omitted from Eq. (15) and 16 assuming that $n \ll N$. In the following step, 100 different values of $c \in [0, \pi]$ (authors' recommendation [27]) are chosen and for every c , the parameter K_c is calculated either by means of linear regression or correlation. We chose correlation method (Eq. (17)):

$$K_c = \frac{\text{cov}(n, D_c(n))}{\sqrt{\text{var}(n) \text{var}(D_c(n))}} \quad (17)$$

$\text{var}()$ is the variance in $\text{cov}()$ the covariance of the argument. In the end, the parameter K is calculated as the median value of K_c (Eq. (18)), with $K \approx 0$ indicating a regular and $K \approx 1$ a chaotic process [27].

$$K = \text{median}(K_c) \quad (18)$$

Resampling of the original time series is advised to avoid oversampling and thus improve the accuracy of the 0-1 algorithm [27, 28]. For the original time series (sampling time $\tau = 1$) we calculated $K = -0.29$ while for the resampled time series ($\tau = 6$) we calculated the value $K = 0.96$, indicating that the process we investigate is indeed chaotic.

5 Conclusions

Mineral wool melt fiberization was studied on an industrial spinning machine. Images of the process were recorded with a high speed camera and analyzed numerically to determine melt film circumferential velocity. Velocity time series was filtered and analyzed employing linear and nonlinear methods. The emphasis was placed on the nonlinear time series analysis using dimensionality, stationarity, chaos and determinism tests in addition to recurrence plots and recurrence quantitative analysis.

Melt fiberization process was determined to be low-dimensional and chaotic by means of the false nearest neighbor algorithm and 0-1 test, respectively. Cross prediction errors and recurrence analysis suggest that the process is nonstationary but is probably composed of both stationary and nonstationary mechanisms. The former are likely to be linked to the periodic phenomena such as the spinner wheel rotation and vibrations while the latter may be associated with the more complex, aperiodic phenomena such as the interaction between the melt string and the melt film or the tearing of fibers from the film. In addition to the aforementioned tests, we also tested the time series for determinism and found that while fiberization process certainly exhibits a form of weak determinism, it cannot be characterized as typically deterministic.

Finally, we can conclude that mineral melt fiberization is definitely a very complex process, the properties of which justify the use of nonlinear time series analysis. However, low dimensionality indicates a possibility for the process dynamics to be modeled mathematically. In addition to the numerical and experimental modeling of the process, a nonlinear analysis-based mathematical model could greatly improve the understanding of the fiberization process and thus allow for further improvements in the spinner operation control and consequently, the quality of the end product.

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Nomenclature

A	grayscale level (brightness) of an 8-bit image, -
D	diffusion coefficient, m ² /s
e	cross prediction error, -
$\mathbf{e}_p$	unit vector (trajectory direction), -
f_0	spinner wheel rotational speed, Hz
fnn	fraction of the false nearest neighbors
I	mutual information, -
K	0-1 chaos test parameter, -
M_{ij}	recurrence in a recurrence plot, {0, 1}
M	determinism test box size, -
m	embedding dimension, -
n	number of trajectory passes through a box in determinism test, -
p_c, q_c	translation variables in 0-1 chaos test
R	spinner wheel radius, m
R_{xx}	autocorrelation function, -
s	ligament spacing on the melt film, m
s_n	delay (phase space reconstruction) vector
t	time, s
Δt	image sampling time, s

U	neighborhood
$\mathbf{V}_k$	average directional vector for k-th box, -
$\mathbf{v}$	absolute velocity vector, m/s
v_x, v_y	absolute velocity of melt film in x and y direction, m/s
We	Weber number, -
Δxy	spatial displacement between consecutive images, pix

Greek letters

ε	neighborhood size
τ	phase space reconstruction time delay, -
κ	determinism factor, -
λ	wavelength, m
μ	melt viscosity, Pa·s
ρ	melt density, kg/m ³
σ	melt surface tension, N/m

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Figure Captions List

- Fig. 1. Simplified presentation of a spinner wheel and initial fiber formation
- Fig. 2. Four-wheel industrial spinner for mineral wool production [9]
- Fig. 3. Experimental setup for visualization of the fiberization process
- Fig. 4. Left: window of interest (yellow rectangle) for velocity calculation. Center: chosen coordinate system. Right: region of interest shown for a sequence of four consecutive frames, 400% magnification.
- Fig. 5. Time series of v_y before (upper panel) and after the application of the low pass filter at 2kHz cutoff frequency (lower panel)
- Fig. 6. Power spectrum of the filtered time series
- Fig. 7. Autocorrelation function and mutual information for the filtered time series of v_y
- Fig. 8. Fraction of false nearest neighbors (fnn) with respect to the embedding dimension m , for $\tau = 19$

Fig. 9. Cross prediction errors for the filtered time series

Fig. 10. Determinism test. The left panel features the reconstructed phase space using $\tau = 19$ and $m = 4$, while the right panel shows the pertaining approximated directional vector field. Determinism factor of the phase space is $\kappa = 0.66$.

Fig. 11. Recurrence plot of the filtered time series

Fig. 12. Recurrence quantification analysis plots

Figure 1 (fig1.tif)

Figure 2 (fig2.tif)

Figure 3 (fig3.tif)

Figure 4 (fig4.tif)

Figure 5 (fig5.tif)

Figure 6 (fig6.tif)

Figure 7 (fig7.tif)

Figure 8 (fig8.tif)

Figure 9 (fig9.tif)

Figure 10 (fig10.tif)

Figure 11 (fig11.tif)

Figure 12 (fig12.tif)