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Abstract:

The design and development of the acoustic eye model (AEM) is reported. The model consists of a computer based simulation that describes the propagation of mechanical disturbance inside a simplified model of a human eye. The capabilities of the model are illustrated with examples, using different laser-induced initial loading conditions in different geometrical configurations typically occurring in ophthalmic medical procedures. The potential of the AEM is to predict the mechanical response of the treated eye tissue in advance, thus complementing other preliminary procedures preceding medical treatments.

1. Introduction

Like all medicine, the field of ophthalmology is developing constantly. In the last decades, with the development of various laser applications a substantial qualitative and revolutionary progress was made in ophthalmic diagnostics, therapy and imaging techniques [1]. Refraction-correcting eye laser procedures as well as photoionization-based therapeutic laser eye surgery such as iridotomy, capsulotomy and vitreolysis have become standard methods of treatment. Lasers are believed to have significantly improved the therapy and surgery outcome and reduced the loss of vision from major causes of blindness [1].

Laser interaction with matter results in heating and subsequent mechanical displacement, the effects ranging from low amplitude ultrasonic waves to more intense phenomena such as explosive boiling and dielectric breakdown accompanied by laser ablation and shock wave formation [2,3]. The desired, primary effects of laser treatment of the tissue, such as cutting, coagulation by heating and photodisruption, are accompanied by other phenomena, mostly in the form of mechanical response of the tissue. These secondary effects are not wanted and may result in diminishing the quality of the treatment outcome, in extreme cases by causing unnecessary degradation and tearing of the tissue. On the other hand, the resulting mechanical waves also carry information about the position, the size and the nature of their source [4,5], so they can be exploited for the monitoring of the medical procedure as well as an assessment tool for the procedure outcome [6].

Due to the interference and refocusing of the propagating waves, the adverse mechanical effects can, depending on the geometry, occur even relatively far away from the laser-tissue interaction site. Many aspects of the mechanical characteristics of the human eye have not been extensively explored yet, although this knowledge would be highly desirable in the search for improvement of medical treatments. Because of its spherical shape, the eye is inherently inclined to focus the reflected ultrasonic waves, resembling the case of focusing the shock waves in lithotripsy medical procedure [7]. Due to similar acoustic impedances of different tissues, the amplitudes of the waves which are reflected from boundaries between tissues are small compared to the amplitudes of the waves, reflected from the free frontal boundary. It would be advantageous for any particular application to know the mechanical response of the treated eye tissue in advance, without having to make lengthy experimental investigations.

Since, to the best of our knowledge, no comprehensive attempt has yet been made to describe the propagation of these laser-induced mechanical waves inside the human eye, a computer-based numerical model was built to simulate the mechanical (acoustic) response of the eye to a range of stimuli. A reliable prediction of the evolution of the laser-induced mechanical displacements is advantageous, because it can reduce the need for certain measurements and procedures which have to be performed before the treatment.

Laser-induced shock waves have already been numerically modelled in solid structures. In [8], laser shock peening process was analyzed and phase transformation from austenite to martensite due to shock waves propagation has been studied in austenitic stainless steel. From the numerical aspect, there are many similarities to this contribution, where the development and the results of the acoustic eye model (AEM) are discussed. Using

the results of AEM simulations, both therapeutic and possible undesirable side effects of laser treatments that are the result of mechanical response of the tissue can be predicted and analyzed.

High benefit is expected from the predictions of AEM simulations in determining, for example, the optimal treatment parameters such as the energy and duration of laser pulses or in finding the safe distance of the laser focus from the lens and the retina so that unnecessary injury can be avoided. Determination of the optimal conditions with the results of the AEM simulations could be used in the standardization of certain medical procedures. Other possible applications of the AEM include simulations of mechanical waves created by mechanical impacts on the eye or explosions in the proximity of the eye [9,10].

2. Laser-induced ultrasound inside the human eye

In various laser based medical procedures, laser energy is concentrated at different locations along the optical axis inside the human eye: at the lens (capsulotomy), inside the vitreous humor (vitrectomy) or at the fundus of the eye (retinal therapeutic procedures). Focusing of a pulsed laser beam results in locally and temporally high energy input causing rapid heating of the tissue, followed by thermoelastic expansion and/or dielectric breakdown. The breakdown plasma is rapidly expanded, a bubble is formed and shock waves that propagate away from the irradiated site are created during breakdown and subsequent bubble collapses. The incoming laser pulse energy is distributed between the shock wave and the bubble in various proportions [11]. The therapeutic effects but also the unwanted damage can be the result of both channels of energy distribution as well as due to the localized initial plasma.

The laser-induced bubble expands and subsequently collapses after reaching its maximum size. The expansion and the collapse phase can be repeated, often in several cycles [12,13]. The dynamical bubble formation process finally leaves behind a number of small bubbles of various sizes near the interaction volume [13–15] which typically stay present for several seconds and more. If a tension pulse propagates through the volume hosting the residual bubbles, they will expand and when the amplitude of the negative pressure is high, the process can be quite intense.

The pressure amplitude of the shock front is decreased as it propagates with supersonic velocity radially away from the bubble. In aqueous and similar media, the wave transforms into an acoustical wave in a few hundreds of micrometers [3,16]. Typically, the dominant frequency content of these waves is in the MHz range [3]. When the wave is propagating within the spherically shaped human eye, our simulation shows that the portion of the wave energy that is reflected at the free frontal boundary of the eye is focused because of the curvature of the cornea and the visible part of the sclera. The region where the energy of the reflected wave is concentrated extends along the symmetry axis of the eye from the center of the lens well into the interior of the vitreous humor. The wave amplitude here is comparable to the amplitude a few millimeters away from the source. Upon reflection at the free boundary, the initial compression pulse is transformed into a rarefaction. Having a relatively high amplitude, the pressure of the rarefaction pulse is well capable of substantially expanding the residual bubbles which exist in the volume [17], leading to possible unwanted damage of the tissue. Microscopic bubbles can even be formed anywhere along the propagation path of the laser beam where a portion of the incoming light is absorbed at impurities within the eye [14,15]. Because of the focusing effects, a seemingly benign acoustic pulse which is expected to have low amplitude because it has already propagated for a few centimeters can become the source of unwanted damage to the tissue of which the surgeon/operator is not even aware of. Even when the expansion of the residual bubbles does not carry enough energy to damage the tissue, the cross-section of the initially small absorbing sites for the incoming light is now greatly enlarged, leading to larger scattering and to the reduction of the beam quality of subsequent laser pulses.

Because of the geometrical considerations, the described effects should be less pronounced when the source of the wave is well off the symmetry axis but very often the laser based procedures are performed at or near the symmetry axis of the eye.

3. Acoustic eye model (AEM) and the numerical procedure

The eye is a complex structure consisting of numerous tissues (Fig. 1) with very different optical but relatively similar mechanical properties. In our acoustic eye model, we consider different tissues of the eye as isotropic, homogeneous and linear elastic, as well as virtually incompressible (Poisson's ratio close to 0.5), all of which is a simplification from the actual behavior. In the present analysis, only the fundamental parts of the eye are considered and geometric simplifications are made, for example the eye is modeled to have axial symmetry.

An axisymmetric finite element model is used to calculate the displacements U_i and pressures P_i within the eye induced by a laser source and to simulate wave propagation. The analyzed geometric domain is composed of eight types of eye tissues, as shown in Fig. 1. Vitreous and aqueous humor are modeled as liquids, having properties of water at 35 °C. Cornea, sclera, lens, ciliary zonule and ciliary body are modeled as virtually incompressible homogeneous and isotropic solids. The fat tissue surrounding the eye is modeled as a semi-infinite

virtually incompressible solid domain in order to simulate non-reflective boundary conditions. Material properties were obtained from the available literature, mainly from the ultrasonic investigations of the human eye, and are listed in Table 1.

Table 1 Material properties of eye tissues

Solid (homogeneous and isotropic)					
Tissue	Density [kg/m ³]	P-wave velocity [m/s]	Elastic modulus [MPa]	Poisson's ratio	Acoustic impedance [kg/(m ² ·s)]×10 ⁶
Cornea	1087 ^g	1559 ^b	20.1423 ^c	0.498680 ^{b, c, e}	1.695 ^c
Sclera	1050 ^c	1608 ^b	25.5082 ^c	0.498680 ^f	1.688 ^c
Lens	1040 ^d	1631 ^b	20.9945 ^c	0.498680 ^f	1.631 ^c
Fat	950 ^d	1450 ^d	15.7637 ^c	0.498680 ^f	1.298 ^c
Ciliary body	1065 ^d	1580 ^d	20.9827 ^c	0.498680 ^f	1.683 ^c
Ciliary zonule					
Liquid (acoustic medium)					
Tissue	Density [kg/m ³]	Wave velocity [m/s]	Bulk modulus [GPa]	Acoustic impedance [kg/(m ² ·s)]×10 ⁶	
Vitreous humor	994.1 ^a	1503 ^b	2.24568 ^c	1.494 ^c	
Aqueous humor					

^a same as water at 35 °C [18]; ^b ref. [19]; ^c calculated; ^d ref. [20]; ^e ref. [21]; ^f same as cornea; ^g ref. [22]

The geometry of the axisymmetric numerical model is illustrated in Fig. 1. For tissues that are considered as homogeneous and isotropic linear elastic solids, the formulation includes Navier-Lamé equations from elastodynamics, involving equilibrium equations, strain-displacement relations and constitutive equations of linear elasticity. Liquid tissues are considered as inviscid and compressible acoustic media, where the dynamic pressure is the only variable and is derived from equilibrium equations, equation of continuity and constitutive equation that considers proportionality of pressure and volumetric strain with bulk modulus being the coefficient of proportionality. Interaction between the acoustic and the solid media is modeled via consistency condition, where pressure at the acoustic medium boundary equals to the normal surface traction loading at the solid medium interface. In current analyses, acoustic attenuation has been neglected.

Two different initial temporal pressure distributions giving rise to mechanical displacements were considered (Fig. 2): (i) a short localized pressure pulse originating within the vitreous humor and representing the temporal pressure distribution of the shock wave induced during the photoionization or bubble collapses and (ii) rapid isotropic expansion at the retina, representing the thermoelastic wave which is launched during the laser-heating of the retina [6].

In order to simulate the source at different positions, three different locations are chosen, where temporal pressure distribution is prescribed as a single nodal loading in vitreous humor. The locations of the loading sources S_i (Fig. 1) are chosen to represent typical situations that arise in different medical applications. Position of the source S_1 at the center of the vitreous humor is indicative of situation at vitreolysis procedure, position S_2 in the vicinity of the lens describes the situation at capsulotomy while the position S_3 in the vicinity of retina is typical of retinal photocoagulation procedure.

Both temporal pressure distributions $P(t)$ are prescribed the same peak pressure $P_{\max} = 10$ MPa, corresponding to the creation of a laser-induced bubble with a maximum diameter of about 1 mm [14], and the same duration of $t_{\text{end}} = 1$ μs in all the presented analyses for easier comparison of both scenarios. In all cases, wave propagation is followed for 20 μs . Regarding the initial peak pressure, it should be noted that in most practical cases, the pressure amplitude of the mechanical waves generated by the thermoelastic source are several orders of magnitude smaller, yet still detectable [6].

Similarly, the choice of the pulse duration (1 μs) serves the purpose of better visualization of the results. Namely, with a typical few-nanoseconds-long therapeutic laser pulse, the resulting duration of the initial shock wave is a few hundreds of nanoseconds [14], but such a short wave would only be visible as a sharp spike on the scale of 20 μs long propagation time and the information about the shape of the wave would not be adequately visualized.

Fig. 1 The modeled domain of eye tissues (left), the r and z coordinates for the selected points (middle) and the details of the axisymmetric numerical model (right).

Fig. 2 Normalized monopolar temporal pressure distribution of laser-induced source following photoionization (blue) and thermoelastic expansion (orange)

As for the pressure and displacement amplitudes, the presented results are scalable within the presented framework of linear elastodynamics. However, the enlarged spatial width of the wave could partly play a role in the formation of the interference pattern after the reflection of the wave. But in that respect, the width of the wave was still taken short enough so that the geometry of the reflecting boundary keeps playing the dominant role.

All the wave propagation analyses are performed in Abaqus/Standard software using implicit dynamic time stepping. In order to minimize numerical oscillations, very small time increments need to be used, therefore the time step is limited to maximally 1 ns. In the same regard, spatial discretization is also important, thus the whole domain is discretized into finite elements having a typical dimension in the order of 50 μm . Namely, the numerical fluctuations are especially difficult to avoid near the symmetry axis at $r = 0$, so extremely high temporal and spatial discretization is required to minimize the undesirable numerical oscillations. Acoustic medium is discretized by 8-node quadratic acoustic finite elements ACAX8, and solid tissues are modeled with 8-node quadratic hybrid elements with linear pressure approximation. The outer edge of the fat tissue elements is connected to four-node axisymmetric continuum one-way infinite elements CINAX4 to model the non-reflective boundary conditions.

Regarding the variables used in the presentation of the results in the following section, two points should be made. Firstly, wave propagation will be presented using frames of the pressure field. In the acoustic medium, the pressure is a primary variable, whereas in the solid it is defined as the negative of the hydrostatic stress, which is one third of the first stress tensor invariant $p = -(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})/3$. Thus, different variables are presented in different types of media on the same contour plot so that the whole domain can be depicted at once.

Secondly, the numerical results will be observed through time and plotted in graphs. In Fig. 1, points S_i are marked as sources of laser-induced pressure, points P_i are the locations where the pressure will be observed, and points U_i are the locations where the displacements in the direction normal to wave propagation will be presented. It should be noted that the displacements cannot be followed in the acoustic media, because there is no mechanical or kinematic variable in the formulation of the acoustic medium. The terms “pressure” and “displacement” will be used in accordance with this explanation throughout the following section. The normal displacement at the free

boundary of the eye is of key interest since it is a measurable observable [6] and by comparing the results of the simulation to the experiment both can be validated.

4. Results and discussion

First, we consider the results of the pressure field evolution. To make the comparison between the different cases easier, the discrete color scale for all the presented cases is the same. According to the legend at the bottom of Figs. 3-5, the positive values of the pressure (compression) are shown in warm colors while the negative values (tension) are shown in cool colors.

In Fig. 3, the propagation of the pressure pulse and its evolution is presented in $1\ \mu\text{s}$ steps for the photoionization source S_1 , located in the center of the eye. As expected, the initially high amplitude of the compression pulse decreases as the wave spreads spherically through the vitreous humor (frames from $1\ \mu\text{s}$ to $4\ \mu\text{s}$). In the interval from $4\ \mu\text{s}$ up to $7\ \mu\text{s}$, the part of the wave that propagates toward the front of the eye is reflected at the different tissue boundaries which it encounters (lens, aqueous humor and cornea). The reflections at these boundaries are not very pronounced because these tissues have similar acoustic impedances Z (see Table 1) and are not even visible on the discretized scale. The relative amount of the reflected acoustic energy R in the case of normal incidence from medium 1 to medium 2 is determined namely by the acoustic reflection coefficient $R = \left((Z_2 - Z_1) / (Z_2 + Z_1) \right)^2$. Here Z_1 is the acoustic impedance of the first and Z_2 of the second medium. At boundaries which the wave encounters on its path along the symmetry axis within the eye, the reflection coefficients for normal wave incidence are a few per mille, in the range from 0.2 % to 0.4 %, while in the perpendicular direction they are in the range from 2 % to 4 %. At the boundary with air in the front of the eye the reflection coefficient is 99.9 %.

The front of the eyeball has smaller radius of curvature than the rest of the eyeball (see Fig. 1) and the part of the wave that is reflected there is focused at a different location and sooner than the contributions propagating through the rest of the eyeball. Starting from the frame at $8\ \mu\text{s}$, the reflections at the front of the eye can be observed as rarefactions (blue wavefronts). At the free boundary, the phase of the reflected wave is reversed compared to the phase of the incoming wave and a rarefaction pulse is now traveling inward. The reflected waves interfere constructively to form a focal volume inside the eye lens (Fig. 3 at $11\ \mu\text{s}$). The location of that point is approximately at half the radius of curvature of the front part of the eye, consistent with the known result from geometrical optics for the focus of a concave spherical mirror. The focal volume can be seen in the next frames to be travelling along the symmetry axis toward the back of the eye.

The evolution of the shape of the wave, which transforms from a half-cycle, monopolar rarefaction pulse to a full single-cycle, rarefaction-compression wave when it passes through the focus, is indicative of the Gouy phase shift effect which was also observed by other authors in the case of focusing of surface and bulk ultrasonic waves in various configurations [23–29]. The Gouy phase shift is a well-known $n\pi/2$ axial phase shift experienced by any converging wave when it passes through a focus [30]. For a two dimensional (cylindrical) wave $n = 1$ and for a spherical wave $n = 2$. In cylindrical geometry, the effect of Gouy $\pi/2$ phase shift when applied to a half-cycle (monopolar) pulse is to broaden the pulse by approximately a factor of 2 and transform the shape into a single-cycle (bipolar) one, as in [23]. On the other hand, the effect of Gouy phase shift of π which occurs in spherical geometry, when it is applied to a half-cycle (monopolar) pulse, is to simply flip the phase, i.e., a rarefaction pulse becomes a compression.

In our case, the geometry cannot be simply described as purely cylindrical or purely spherical. Namely, a spherical wave is being reflected from the boundaries of an axially symmetric structure. The different contributions of the reflected wave that interfere constructively on the axis to form a focus can be viewed as originating from a series of ring sources at the front of the eye, explaining the apparent $\pi/2$ shift (Fig. 3) of the wave. As the following quasi-cylindrical contributions meet and interfere further down the axis towards the center of the eye, so is the resulting focal volume elongated in that direction. The relatively high amplitude is maintained while the wave is propagating toward the center of the eye.

The rest of the initial spherical outward wave which propagates in other directions arrives at the outer border of the vitreous humor at approximately $7\ \mu\text{s}$. It is partially reflected there from the sclera and from the fat tissue but the reflected wave amplitude is again not seen in Fig. 3. The reflected wave can be observed again at $14\ \mu\text{s}$ (double propagation time), partly as a turquoise ring (rarefaction) traveling inward and partly as two compressions inside that ring. The interpretation of the detailed shape of the field is difficult due to many reflections it has experienced. The focusing of that portion of the initial wave takes place approximately at the center of the eyeball at $15\ \mu\text{s}$ when it gets superimposed on the part of the wave coming from the front of the eye described before. The slightly longer path which this wave had to travel is compensated to a certain degree with the higher propagation velocity inside the lens, thus the arrival of the two waves at the focal point almost coincides. In the next frame, at $16\ \mu\text{s}$, an outward compression appears again indicating the existence of another Gouy phase shift, this time dominantly as π phase reversal. The relative amplitudes of these waves will be discussed later on

when waveforms at the selected points are presented. The animation of the described wave propagation is shown in Online Resource 1.

Fig. 3 Pressure field at consecutive times, source at S_1 , in the center of vitreous humor

Fig. 4 Pressure field at selected times, source at S_2 , near the eye lens

Fig. 5 Pressure field at selected times, source at S_3 , at the retina

In Fig. 4, only the selected frames from a similar analysis are shown for the laser-induced source placed in the vitreous in the vicinity of the lens (source S_2 in Fig. 1), simulating wave propagation during the capsulotomy medical procedure. The corresponding animation is provided in Online Resource 2. The description of the wave propagation and superposition is similar to the previous case, with a few differences. The source is now closer to

the front of the eye, approximately in the center of the curvature of the cornea [31]. The radius of curvature of the wavefront arriving at the reflecting boundary is practically the same as the radius of curvature of the boundary itself. Consequently, the reflected waves interfere constructively to form a caustic starting approximately at the location of the source (frames at 8 μs and 9 μs). Again, after passing the focus, the reflected wave exhibits a change in the pulse shape due to the Gouy phase shift and again the wave propagating along the axis is observed to have the maximum amplitude. The focusing of the rest of the wave is not so pronounced, because of the different geometry than in the previous case. Because of the discrete scale, the rarefactional wave in Fig. 4 (frames from 8 μs to 13 μs) appears as having side lobes, but in fact, the lobes are connected to the axial rarefaction. This is best observed in the animation of Online Resource 2.

The pressure field evolution in the third case is presented in Online Resource 3 and illustrated in Fig. 5. In this case, the source is located at the fundus of the eye, close to the retina. Here the temporal distribution of the source is slightly different, corresponding to the laser-induced thermoelastic expansion at the retina. In this case, at the beginning of the propagation, the wave does not encounter any reflecting surfaces head-on and its evolution appears rather uneventful. Represented on the same discretized scale as the two previous cases, the wave amplitude is vanishingly small from the 8 μs to 17 μs frames. But during this time, a part of the wave has propagated to the front of the eye and exhibits the same behavior upon reflection as in the two previous cases: it interferes constructively to form a focus inside the eye lens, which extends further inside the eye, experiencing the Gouy phase shift in the process. In this case, the curvature of the wavefront before the reflection at the front of the eye is much larger than in the previous two cases, more resembling a plane wave. This configuration was experimentally realized by Brinkmann et al. [6].

To give a further insight, the evolution of the pressure and displacement is presented at certain points which are depicted schematically in Fig. 1. The locations of these points are chosen for different reasons:

- Points P_1 and P_2 are set to be at the same distance from the source S_1 : P_2 is on the symmetry axis while P_1 is perpendicular to the symmetry axis, in order to test the consistency of the computed results and on the other hand to observe the differences in the waveforms which occur later on.
- Points P_4 and P_5 on the symmetry axis are chosen to observe the pressure wave at focusing.
- Point P_3 serves as a control point in the middle of the propagation path.
- Points U_1 and U_2 are chosen to show the displacement of the free boundary at the front of the eye: U_1 because it is on the symmetry axis and U_2 because it is of practical importance since it could be compared to actual measurements [6].

In Fig. 6, the pressure waveforms at points P_1 and P_2 are shown for the case of a centrally positioned photoionization source S_1 . Since the distance of both points from the source is the same, the corresponding waveforms coincide during the first passage of the pulse where they exhibit a typical shock wave shape with a sharp rise followed by a less steep tail-like decrease. The two waveforms begin to differ after 13 μs when the reflected waves return to both locations. The wave returning to P_2 exhibits a well-defined phase reversal due to a reflection at the free boundary and also a transformation from an initially monopolar (compression) to a bipolar (rarefaction – compression) pulse shape. A substantially lower amplitude would be expected after the wave has spread spherically for 14 μs compared to the amplitude it has close to the source at 1 μs . But the amplitude at point P_2 is the result of the interference and focusing of the reflected wave due to the geometry, as was already discussed in the description of the pressure field distribution given in Fig. 3.

On the other hand, the wave returning to point P_1 which is located off the symmetry axis (Fig. 6) does not experience polarity reversal since it is not reflected from a free boundary, but rather from a combination of two boundaries in series: first at the vitreous-sclera and immediately after that from the sclera-fat boundary. As discussed above, the amount of the reflected energy here is relatively small because the tissues have similar acoustic impedances. The returning wave is a superposition of the two reflections, the second occurring so close in space and time to the first one that they overlap and the combined wave is spread spatially and temporally accordingly. Regarding the on-axis and off-axis amplitudes of the returning waves, the amplitude of the wave at point P_2 (at 14.5 μs) is -32.5 kPa, having almost the same absolute value as the peak in the first passage of the outgoing wave ($+35$ kPa at 1.0 μs), while the first peak of the reflected wave at point P_1 (at 14 μs) is only $+5$ kPa, more than six times smaller.

Fig. 6 Pressure waveforms at points P_1 and P_2 , source at S_1

Fig. 7 Pressure waveforms at points on the symmetry axis: a) source at S_1 , observation point P_5 , b) source at S_2 , observation point P_4 , c) source at S_3 , observation point P_5

Fig. 8 Displacements of the cornea, source at S_3 , observation points U_1 and U_2

In Fig. 7, the waveforms of pressure evolution are presented for various source positions at points P_4 and P_5 which were found to be of interest because of the focusing effects at these locations. The description of the waveforms follows along the same lines as above. The expected decrease of the primary wave amplitude with distance from the point source is evident, with the primary wave in Fig. 7b occurring the earliest, at 1 μ s, followed by the smaller amplitude wave at 5.5 μ s in Fig. 7a and even smaller one at 13 μ s in Fig. 7c. In all three cases, the reflected wave is observed at a focusing point, having a markedly high amplitude with the initial phase reversed because of the reflection and then transformed to a bipolar pulse due to the Gouy phase shift.

The displacements history of the cornea at points U_1 and U_2 is shown in Fig. 8 for the case of the thermoelastic source at the retina S_3 . These numerical results are important because of the possibility to directly compare the amplitude and the time dependence to actual measurements of the displacements [6]. The distance from the source to point U_2 is slightly smaller than the distance to point U_1 , which is reflected in a shorter wave arrival time at point U_2 . It should be pointed out that in the present analysis, the boundary at the front of the eye is free which is not true when contact measurements of the displacements are performed. In that case, the displacement waveforms should be obtained by rerunning the simulation with the contact lens commonly used in ophthalmological treatments included in the model.

The same is true for the evaluation of amplitude and phase of the reflected pressure wave propagating inside the eye. The laser medical procedures are usually performed with the aid of optics for beam manipulation which is in contact with the front of the eye. This change in boundary conditions can play an important and positive role regarding the undesirable effect of the rarefaction pulse expanding the residual bubbles, described in section 2. Depending on the acoustic impedance of the adjacent material, the reflected wave amplitude can be smaller compared to the present case and the pulse can be reflected without the change in phase, i.e., the compression is reflected as a compression. But even then, because of the Gouy phase shift, a rarefaction will be formed after passing the focus. All these considerations confirm that it is advisable to perform a preliminary comprehensive simulation for each individual case.

5. Conclusion

A computer based acoustic eye model describing the propagation of mechanical disturbances inside a human eye is presented, with simplifications regarding the material properties and the geometry of the eye. As a consequence of the simplifications, its applicability is limited to a certain extent. For example, the pressure and displacement amplitudes of the model are overestimated. The measured amplitudes are expected to be smaller, (i) because the acoustic attenuation losses were not taken into account, (ii) because the cylindrical symmetry causes the waves emanating from the axis to constructively interfere back on the axis again after reflection from the free frontal boundary, and (iii) because of their finite-size, the sensors average out the pressure distribution under their aperture, thus reducing the maximum point-pressure. While the initial loading conditions of the model are arbitrary, the present analysis uses the loading temporal distribution which is characteristic of laser-induced stress.

The analysis of three different examples with different initial conditions and loading are presented and discussed in detail. The result which is common to all three examples is the formation of a mechanical wave caustic inside the eyeball, with its shape and location depending on the initial and boundary conditions. The factors that play a crucial role in its creation and evolution are the curvatures of the eyeball and the boundary conditions which together with the Gouy phase shift define how the waves are reflected, how they interfere and how their shape is transformed. Because of the complex interplay of these influencing parameters it would be helpful to reliably simulate the estimated pressure and displacement field for each particular application in advance. The presented results illustrate the potential uses of the simulation using the described model.

Quantitative comparison of the results of the model to the already published [32,33] and new experiments will lead to further development and improvement. In continuation, based on this comparison, the model can be adjusted and customized for different procedures in ophthalmology. Further refinement of the model consists of several iterations of acquiring relevant experimental data and performing suitable improvements. In perspective, the AEM can also serve as a tool to explore and verify novel ultrasonic and displacement detection techniques.

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