ARTICLE TEMPLATE

Surface-patterning generated half-skyrmion lattices in cholesteric blue phase thin films

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ABSTRACT

Liquid crystalline blue phase II sandwiched between a pair of confining surfaces with specifically designed patterns is studied using numerical simulation based on the Landau-de Gennes phenomenological approach. Affected by the surface patterns, quasi-two dimensional half-Skyrmion lattices form. Specifically we observe the formation of square and triangular lattices of half-Skyrmions. These lattices of the swirling skyrmion textures are interspersed with singular $-1/2$ or -1 line defects which also constitute quasi-two dimensional regular lattices. Finally, this work is a contribution to designing and stabilising complex birefringent layer profiles of various symmetries.

KEYWORDS

blue phases; half-skyrmions; surface patterning; confinement

1. Introduction

Analogies between different physical systems have recently received increasing attention. Among them, analogies between cholesteric liquid crystals and chiral magnets are of particular interest[1–3]. Various topological defects of the same kind have been found both in the two seemingly quite distinct systems. Among the topological objects shared by liquid crystals and magnets, skyrmions are a swirling spin texture without singularities. Although first introduced in the context of nuclear physics[4], they have found use in condensed-matter systems such as chiral magnets[5], liquid crystals[6] and Bose-Einstein condensates[7,8]. In recent years, skyrmions in magnets have been the centre of attention, owing to their prospective applications in information technology such as data storage[5]. In a full-skyrmion, the unit vector order parameter $\mathbf{n}$ lies parallel to the rotational axis of symmetry at its centre, and rotates gradually to change its sign at its perimeter. If $\mathbf{n}$ only rotates by $\frac{\pi}{2}$ from the centre to the perimeter, it is called a half-skyrmion. Note that, in liquid crystals, the director field $\mathbf{n}$ is not vectorial, but linelike, with the equivalence between $\mathbf{n}$ and $-\mathbf{n}$. This feature enables some structures, which cannot be realised in other systems, to form in liquid crystals. One crucial advantage of liquid crystals over other condensed matter systems for studying topological defects is that they are more experimentally tractable, with

no need for extreme experimental conditions. A standard way of generating and stabilising defects in liquid crystals is using confinements. Cholesterics that are confined by homeotropic surface anchoring into layers with thickness smaller than a pitch have a characteristic feature, that is, the director field is homogeneous and unwound. In such frustrated states of cholesterics, individual torons[9] and two dimensional lattices of torons[10,11] can be optically generated. Bistable electric-field-induced torons were found in bubble domains[12]. Depending on surface anchoring, confining cholesterics into spherical droplets leads to various fascinating structures[13–16]. These studies stimulated interest for optical and photonic applications of confined cholesteric liquid crystals.

While the effects of confinements on nematic and cholesteric liquid crystals have been extensively studied, until recently much less has been known about the effects of confinements on liquid crystalline blue phases (BPs)[17]. BPs occur in highly chiral nematics over a narrow temperature range as an intermediate between isotropic and cholesteric phases. There are three types of BPs referred to as BPI, BP II and BP III, appearing in the sequence of increasing temperatures. Unlike BP III, which does not show crystalline symmetries, BPI and BP II possess ideal three dimensional lattice symmetries in the molecular orientational order. The typical structure of bulk BP II is shown in figure 1(a-c). Figure 1(a) illustrates the building block of BP II – the double twist cylinder. Figure 1(b) shows the double twist director alignment in a cross section of the double twist cylinder. Figure 1(c) illustrates one typical BP II unit cell, in which line defects interdisperse the double twist cylinders. In the double twist cylinders, the director rotates by $\frac{\pi}{4}$ from the centre to the perimeter. Thus a double twist cylinder can be regarded as a $\frac{1}{4}$ -skyrmion. [The \$\frac{\pi}{4}\$ -turn of the director in the double twist cylinder results from geometrical frustration – conflicts between the local free energy requirement and the three dimensional packing.](#)

In recent years, stimulated by the discovery of wide temperature range BPs[18,19], behaviors of BPs under geometric confinements have attracted great interest, because of the richness of the confinement-induced structures[6,20–23] and possible real-life applications led by those structures[24–26]. The representative confining geometries are a thin cell composed of two parallel surfaces[6,20–22] and a spherical surface[23]. Instead of confining BPs into closed geometries, one can also disperse colloidal particles into BPs[27–30], which is an alternative way of generating novel BP structures. Between parallel surfaces with homeotropic anchoring, an array of double helix disclination lines and two parallel arrays of undulating disclination lines were predicted for BPI[20,21]; whereas for planar anchoring, an array of isolated ring disclination lines was found[22]. More interestingly, quasi-two-dimensional lattices consisting of half-skyrmions, were predicted numerically in such thin cells[6,31] and corroborated by optical microscopy in thin films[31].

The above-mentioned work on confined BPs mostly used uniform confining surfaces. In Ref. [32], chemically nano-patterned confining surfaces were designed on substrates, which enabled the formation of stable BP monocrystalline domains over macroscopic (millimetre) length scales, and directed the orientations of the crystallographic lattice planes. On a patterned surface, interactions between the surface and the liquid crystal have different forms depending on the regions, thus forming a specific pattern. Aided by patterned confining surfaces, the BPI-BP II transformation was found to be martensitic-like, taking place in a diffusionless manner[33]. It is reasonable to expect that, specifically designed patterned confining surfaces can help discover and stabilise various complex BP structures that could be relevant to optical and photonic applications.

In this paper, we study thin BP layers confined by two parallel surfaces that are equally patterned, imposing distinct planar-homeotropic surface anchoring profiles. With numerical simulations, we demonstrate that, by means of a simple strategy of surface patterning, different quasi-two dimensional half-Skyrmion lattices can be created in thin films of BPII liquid crystals. Two types of surface patterns are considered, imposing effective square and trigonal lattices of the Skyrmion arrangements. The lattices of singular topological defects that interpenetrate skyrmions are also shown, as affected by confinement and actual dimensions of surface pattern patches in comparison to the nematic correlation length. More generally, this work contributes to the approaches for designing and stabilising skyrmion-based structures, as material elements for possible use in photonics, storage or rheology applications.

2. Methods

2.1. Landau–de Gennes free energy minimisation

In our numerical modelling, we employ Landau-de Gennes phenomenological free energy minimisation approach where the bulk free energy density of the system is expanded in powers of the local tensor order parameter Q_{ij} , and its spatial derivatives $\partial_k Q_{ij}$. The bulk free energy density of cholesteric liquid crystalline materials is written as[34]

$$f_b = \frac{A_0(1-\gamma/3)}{2}Q_{ij}^2 - \frac{A_0\gamma}{3}Q_{ij}Q_{jk}Q_{ki} + \frac{A_0\gamma}{4}(Q_{ij}^2)^2 + \frac{L}{2}[(\epsilon_{ikl}\frac{\partial Q_{lj}}{\partial x_k} + 2q_0Q_{ij})^2 + (\frac{\partial Q_{ij}}{\partial x_j})^2] \quad (1)$$

where A_0 is a material constant with the dimensions of the energy density; γ is an effective dimensionless temperature that drives the mesophase transitions; splay, bend and twist deformations of the director field are assumed to be equally energetically costly, thus a single elastic constant L is used; q_0 is the inverse cholesteric pitch, related to the cholesteric pitch length p_0 via $q_0 = \frac{2\pi}{p_0}$; ϵ_{ijk} is the rank-3 Levi-Civita tensor. Summation over repeated indices is implied. The first three terms describe the thermal free energy density, and the last two terms denote the elastic free energy density under single elastic constant approximation. Material constants of cholesteric liquid crystals are chosen as follows: $A_0 = 1.02 \times 10^5$ J/m³ and $L = 2.5 \times 10^{-11}$ N are adopted from Ref. [35]; $\gamma = 3.0$; As done in Ref. [34], a dimensionless parameter κ is introduced to characterize the chirality as $\kappa = \sqrt{\frac{108q_0^2L}{A_0\gamma}}$, and its value is taken to be 1.0. With these chosen material parameters, stable bulk BPII can be obtained.

In the presence of surface interactions, the bulk free energy is complemented by the surface free energy. In this work, we use homeotropic and planar uniform surface anchoring with selected easy directions and strengths in different regions of the surface pattern. In the evaluation of the surface free energy, we adopt the commonly used Rapini-Papoular-like surface free energy density formulation

$$f_s = \frac{1}{2}W(Q_{ij} - Q_{ij}^0)^2, \quad (2)$$

where W denotes the strength of the surface anchoring. We take $W = 10^{-2} \text{ J/m}^2$ ($W = 10^{-7} \text{ J/m}^2$) in regions where strong (weak) anchoring is imposed. Q_{ij}^0 is the surface-preferred uniaxial tensor order parameter and is taken as $Q_{ij}^0 = \frac{1}{2} S_{eq} (3n_i n_j - \delta_{ij})$ where S_{eq} is the nematic order parameter in equilibrium and n_i is the position-dependent surface-preferred director.

To obtain the skyrmion structures, we minimize the free energy functional $F(Q_{ij}) = \int dV f_b + \int dS f_s$ numerically employing the finite difference relaxation method which performs on a cuboid mesh. The finite difference relaxation method is an iterative method, during its process, the tensor order parameter Q_{ij} , as well as the morphology of the liquid crystal system determined by Q_{ij} evolve from the initial configurations towards equilibrium. In our calculations, we first obtain the bulk BPII lattice structure numerically; then the bulk BPII structure is appropriately reshaped and used as the initial configuration in studying the surface patterning effects.

2.2. Bulk blue phase structure simulation

Depending on the initial condition, the structure relaxes to the local or global free energy minimum, corresponding to the metastable or stable structure. To obtain bulk blue phase lattice structures, analytic expressions of Q_{ij} derived in the high chirality limit ($q_0 \rightarrow \infty$)[17] are chosen as the initial configuration. For BPII, they are

$$\begin{aligned} Q_{xx} &\propto \cos(2\pi z/a) - \cos(2\pi y/a), \\ Q_{xy} &\propto \sin(2\pi z/a), \end{aligned} \quad (3)$$

with a the size of one cubic unit cell. Other components of Q_{ij} are obtained via simple cyclic permutations. During the relaxation procedure, general profiles defined by the analytic expressions are preserved, and equilibrium structures (i.e., bulk BP structures) emerge[34]. Considering the periodicities of the bulk BPII lattice, we can limit the calculation to one unit cell, and adopt periodic boundary conditions at the unit cell surfaces. Since the unit cell constant a is unknown a priori, a series of calculations using continuously varying a are performed, and the solution with the lowest free energy density (i.e., the total free energy of the unit cell of some size, divided by the unit cell volume) is picked out as the equilibrium one. For material parameters described in section 2.1, the bulk BPII lattice constant in equilibrium is $a = 360 \text{ nm}$.

2.3. Designing the surface patterns and the initial configurations

The goal of this work is to generate square and triangular half-skyrmion lattices in cholesteric liquid crystals, which exhibit BPII structures in the bulk, by means of surface patterning. In a half-skyrmion with the $\frac{\pi}{2}$ -turn director profile, bend deformation contributes substantially to the elastic free energy in the outer part of the structure. As a result, the free energy density of half-skyrmions is expected to be higher than that of the bulk BPII, in which double twist cylinders have a $\frac{\pi}{4}$ -turn director profile. The role of surface patterns is to induce specific inner tension, thereby stabilising the skyrmion structures with higher free energy density.

Hereafter, all length quantities are dimensionless, rescaled by bulk BPII lattice constant a . We assume that the upper and lower surfaces (separated by a distance d) confining the chiral nematic layer are equally patterned. We use four different surface patterns called Pattern S1, Pattern S2, Pattern H1 and Pattern H2, the unit

cells of which are illustrated in figure 2(a-d). Pattern S1 and S2 possess the square symmetry (with the lattice constant b) and are used for generating square skyrmion lattices, whereas Pattern H1 and H2 possess the hexagonal symmetry (with the lattice constants b and $\sqrt{3}b$) and are designed for generating triangular skyrmion lattices. Pattern S1 and H1 are both composed of narrow strips (shown in red) with the width t , and circular spots (shown in light blue) with the diameter t . Pattern S2 (Pattern H2) is obtained by removing the narrow strips in Pattern S1 (Pattern H1). The circular spots are arranged in a square lattice in figure 2(a-b) and a triangular lattice in figure 2(c-d). The narrow strips lie along the perpendicular bisectors of the edges of the squares (figure 2(a)) or triangles (figure 2(c)). In the narrow strips, surface-preferred director orientations are chosen uniformly planar as denoted with the black double-headed arrows, and strong surface anchoring is used; in the circular spots, strong homeotropic surface anchoring is imposed; whereas in regions outside the strips and spots, weak homeotropic anchoring is imposed.

In the simulation, the initial configurations are designed to have the same symmetries as the patterns. For Pattern S1 and S2, the original bulk BPII is uniformly compressed or dilated in x and y directions to fit the patterns. The green netted circular and rectangular regions in figure 2(a-b) represent regions where the patterned surfaces intersect the double twist cylinders. The green netted circles are concentric to the light blue circles. For Pattern H1 and H2, the original bulk BPII is first distorted to have the hexagonal symmetry – it is compressed by a factor of $\sqrt{3}/2$ along y direction and sheared by $\pi/6$ along x direction, such that double twist cylinders lying along z axis are arranged in an effective triangular lattice; then this distorted bulk BPII is uniformly compressed or dilated in x and y directions to fit the patterns. After the distortion, the cross sections of double twist cylinders lying along z axis become elliptical (see the green netted ellipses in figure 2(c-d)). To fit the distance d , the original or distorted bulk BPII is also compressed or dilated along z axis. In the four lateral surfaces of the simulation box, periodic boundary conditions are adopted. During the numerical relaxation process, line defects are expected to form. The gray rings in figure 2 denote locations around which the expected line defects intersect the confining surfaces. We do calculations with different d and b to study their influences on the confined BP structures and find the optimal packing of skyrmions.

3. Results

3.1. Pattern S1

The structure of director field and disclinations obtained using Pattern S1 with $d = 0.3$, $b = 1.2$, $t = b/36$ are shown in figure 3(a). Disclinations are drawn (in yellow) as isosurfaces of the scalar order parameter with $S = 0.2$. Director is shown (in green) in the midplane of the simulation box. 2×2 unit cells are shown. At the regions where four planar strips (rectangles in red) in Pattern S1 meet, there is a pair of $-1/2$ disclinations separated by a short distance (approximately $1/4$ of the unit cell edge length) along the y direction. This pair of disclinations approximately lie in the xz plane, and the angles they make with the z axis are the opposite of each other. The isosurfaces of $S = 0.3$ are shown in red. The pair of separated isosurfaces of $S = 0.2$ merge into one single isosurface with larger S , implying the possibility of realising -1 disclinations. Between the pair of disclinations, the director field is nematic-like. From the director field, typical half-skyrmion features can be clearly seen. The centre of each

half-skyrmion coincides with the centre of one homeotropic spot (light blue circle) in Pattern S1 in the xy coordinates. In the centre of each half-skyrmion, the director points along the z direction. It rotates by $\pi/2$ from the centre of the half-skyrmion to the perimeter. These half-skyrmions constitute a square lattice.

In order to investigate the effects of the thickness of the planar strips and the homeotropic spots on the confined BP structure, we do calculations with $t = b/12$ keeping d and b unchanged. The obtained structure is shown in figure 3(b) where disclinations are drawn (in yellow) as isosurfaces of the scalar order parameter with $S = 0.12$. The free energy density becomes about 11% higher because of the stronger confinement. In this case, half-skyrmions still form a square lattice. However, due to larger t , each pair of $-1/2$ disclinations shown in 3(a) merge into one single -1 disclination which is perpendicular to the patterned surfaces. Each half-skyrmion is an ideal square. The -1 disclinations lie exactly at the vertices of the squares, coinciding with locations where four planar strips meet in Pattern S1. Therefore, disclinations also constitute a square lattice, being the dual lattice of the half-skyrmion lattice. Influenced by the patterned surface anchoring, the isosurfaces of $S = 0.12$ are thinner near the patterned surfaces and thicker away from the patterned surfaces, deviating a lot from cylindrical. A four-fold symmetry can be seen, owing to the four-fold symmetry imposed by the planar strips.

Figure 4(a) shows the results obtained using Pattern S1 for $d = 0.5$, $b = 1.2$, $t = b/36$. Pairs of disclinations are also observed. However, each pair of disclinations is separated along x direction, by a relatively larger distance (about $1/3$ of the unit cell edge length). The skyrmion lattice is still square, and effectively, perimeter of each half-skyrmion form a rectangle.

Figure 4(b) shows the half-skyrmion lattice obtained using Pattern S1 for $d = 1.0$, $b = 1.2$, $t = b/36$. Disclinations become very curvy. Each pair of disclinations are separated along the diagonal direction of the midplane of the simulation box. The director field is locally uniform not only between each pair of disclinations but between any two $-1/2$ disclinations.

When d is increased up to 1.3, the typical skyrmion profile is not observed anymore, and a different structure emerges, as shown in figure 4(c). Apart from disclinations that connect the two confining surfaces, we observe spiral-like disclinations that nearly lie in the midplane of the simulation box along the y direction.

The free energy density (i.e., the total free energy divided by the volume of the simulation box) as a function of d and b in the case of Pattern S1 with $t = b/36$ is shown in figure 5. When the cell is very thin ($d \leq 0.5$), The free energy density first drops with the increase of b . After achieving the minimum, the free energy density starts to increase with the increase of b . For b ranging from 0.8 to 2, half-skyrmion lattices can always be obtained. For thicker cell ($d \geq 0.7$), the free energy density also first drops with the increase of b . However, after achieving a local minimum, the curve becomes non-monotonic, owing to the structural transitions (denoted with dotted lines) with the increase of b . Therefore, for thicker cell, regular lattices composed of half-skyrmions and disclinations cannot form with large b . The structure shown in figure 4(c) is such an example.

3.2. Pattern S2

Using Pattern S2 with $d = 0.3$, $b = 1.2$, $t = b/36$, a more complicated half-skyrmion lattice can be obtained, as shown in figure 3(c). 2×2 unit cells are shown. The centres

of the half-skyrmions still form a square lattice. Along the diagonal direction of the squares, pairs of $-1/2$ disclinations are separated by a distance [slightly larger than \$1/4\$ of the diagonal length of the square](#). These disclinations are perpendicular to the confining surfaces, and form a distorted honeycomb lattice. The free energy density of this structure is lower than that of the structure shown in figure 3(a), implying that this structure is more stable.

In figure 6(a), we shown the half-skyrmion lattice obtained using Pattern S2 with $d = 1.0$, $b = 1.2$, $t = b/36$. The profile is close to that obtained using Pattern S1 with the same parameters (i.e., the profile shown in figure 4(b)), but the defect lines are less curvy. The free energy density of this structure is lower than that of the structure shown in figure 4(b).

Increasing d up to 1.3, the obtained structure (shown in figure 6(b)) is quite similar to that obtained using Pattern S1 with the same parameters (i.e., the structure shown in figure 4(c)). Not only disclinations connecting the confining surfaces but also spiral-like disclinations that nearly lie along the y direction in the midplane of the simulation box are obtained. This means that, for thicker cell, planar strips in Pattern S1 do not have any significant role.

3.3. Pattern H1

Figure 7 shows the skyrmion profile obtained using Pattern H1 with $d = 0.3$, $b = 1.4$, $t = b/36$. 1×2 unit cells are shown. In this structure, half-skyrmions form a triangular lattice. Whereas $-1/2$ disclinations form a honeycomb lattice which is the dual lattice of the triangular skyrmion lattice. The whole structure possesses a hexagonal symmetry. For $b = 1.4$, the hexagonal structure is stable up to $d = 0.8$, whereas beyond this the hexagonal symmetry disassembles. As an example, we show the structure obtained with $d = 1.0$ in figure 8(a). Again, we observe two types of disclinations: the first type connect the two patterned surfaces, and the second type lie nearly parallel to the patterned surfaces.

The free energy density computed as a function of d and b in the case of Pattern H1 with $t = b/36$ is shown in figure 9. Similar to the case of Pattern S1, for thinner cell, the triangular half-skyrmion lattice is stable for large range of b . For thicker cell, structural transitions ([denoted with dotted lines](#)) can take place with changing b , which results in the non-monotonicity of the curves.

3.4. Pattern H2

From figure 9, we can see that the free energy density of the triangular skyrmion lattice can be quite low, comparable to that of the bulk BPII which is -568.9 J/m^3 , indicating that these structures can be energetically very stable. Indeed, without the aid of the planar strips, only using Pattern H2 with homoetropic spots, we can already obtain the same triangular half-skyrmion lattice as that shown in figure 7, provided that the values of d and b corresponds to lower free energy density in curves shown in figure 9.

For Pattern H2 with $b = 1.4$, the triangular skyrmion lattice structure is stable until $d = 0.9$. When $d = 1.0$, the structure transforms into a new lattice structure shown in figure 8(b). [A similar structure was previously observed in a simulation without surface patterns](#)[6]. This structure almost possesses a hexagonal symmetry, but is composed of two types of disclinations. One type of disclinations are nearly perpendicular to the patterned surfaces, and form a triangular lattice; the other type of disclinations

make four-arm junctions, in which one arm is nearly perpendicular to the patterned surfaces, whereas other three arms make the same angle with each other and are not in contact with the patterned surfaces.

For some values of d and b , planar strips (i.e., Pattern H1) are a necessity for realising the triangular skyrmion lattice. For instance, using Pattern H1 with $d = 0.3$, $b = 2.0$, $t = b/36$, one can obtain the triangular skyrmion lattice structure like the one shown in figure 7. However, using Pattern H2 with the same parameters, a different lattice structure emerges, as shown in figure 10. It is a regular lattice but without the hexagonal symmetry. The typical skyrmion profile is barely seen either.

3.5. Discussion

In Ref. [6] and [31], half-skyrmion lattices are generated in BP thin films by homogeneous confining surfaces (without patterns). Their periodicities of the confined BPs are formed naturally as imposed by the bulk BP material. In experiments, the liquid crystal in the isotropic phase is cooled down, and periodic structure with the optimal lattice constant forms spontaneously. In our work, there is an energetic interplay between the periodicity of the surface pattern and the inherent pitch of the blue phase material – that roughly determines the actual size (diameter) of the skyrmions. We chose the skyrmion size and surface pattern periodicity to be compatible; however, if these characteristic scales were chosen as substantially different, then other unexpected non-regular structures could emerge. Further on, with differently designed surface patterns, different types of skyrmion lattices as well as other types of periodic structures are expected to be possibly realised.

The half-skyrmion lattices predicted in our work show interesting [analogies and differences between liquid crystal and magnetic systems](#). In magnets, the unit vector order parameter $\mathbf{n}$ (effectively, the magnetisation) is genuinely vectorial. [This feature allows the formation of square half-skyrmion lattices, but prohibits the formation of triangular half-skyrmion lattices, in which case \$\mathbf{n}\$ is undefined if \$\mathbf{n}\$ at the perimeter of the skyrmions always lies in the two-dimensional lattice plane.](#) In contrast, the director $\mathbf{n}$ in liquid crystal systems is linelike, with the head-tail symmetry. This distinctive feature allows [not only square but also triangular half-skyrmion lattices to form in confined highly chiral nematic liquid crystals.](#)

4. Conclusion

Using numerical modelling, several types of half-skyrmion lattices are shown to form in BPII thin films confined between a pair of parallel surfaces with specifically designed patterns. The surface patterns are composed of patches on which different types of surface anchoring (i.e. nematic field directions) are imposed. Specifically, we show that surface patterns with different symmetry lead to different lattices of half skyrmions, including square and triangular. We observe that the half-skyrmions elastically adapt their profiles, effectively getting shrunk or expanded in distinct directions to cover square, hexagonal or rectangular regions. Skyrmion lattices are shown to be interpenetrated by lattices of singular nematic defects, that reflect the symmetry of the skyrmion organisation. [Our approach can be extended to BPI thin films, with the possibility of observing more types of skyrmion lattices in addition to the square and triangular lattices.](#) Our work contributes to the surface patterning approaches for designing liquid crystal structures that may find use in practical applications, such as

photonic or storage devices.

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Figure 1. (a) Double twist cylinder – the building block of BPII. (b) Double twist alignment of the director (in green) in a cross section of the double twist cylinder. This alignment can be regarded as a $\frac{1}{4}$ -skyrmion. (c) One unit cell of BPII including the double twist cylinders (in blue) and disclination lines drawn as isosurfaces (in yellow) of nematic degree of order $S = 0.16$. (d) A pair of parallel patterned surfaces (in blue) separated by a distance that equals 0.3-1.3 times the bulk BPII unit cell size, between which the BP liquid crystal is confined.

Figure 2. Four different surface patterns of the confining surfaces. (a-b) Patterns for generating square half-skyrmion lattices. 2×2 unit cells are shown. (c-d) Patterns for generating triangular half-skyrmion lattices. 1×2 unit cells are shown. Green nets represent regions where the confining surfaces intersect the double twist cylinders; director orientations are imposed perpendicular to the confining surfaces by the strong uniform surface anchoring in regions denoted by circles (in light blue) with diameter t , and parallel to the confining surfaces in regions denoted by narrow strips (in red) with width t inside which the double-headed arrows (in black) denote the surface-preferred director orientations; gray rings denote locations around which singular defects are expected to form.

Figure 3. (a) Square skyrmion lattice obtained using Pattern S1 with $d = 0.3$, $b = 1.2$, $t = b/36$. (b) Square skyrmion lattice obtained using Pattern S1 with $d = 0.3$, $b = 1.2$, $t = b/12$. (c) Square skyrmion lattices obtained using Pattern S2 with $d = 0.3$, $b = 1.2$, $t = b/36$. Disclinations form a distorted honeycomb lattice. Disclinations are drawn (in yellow) as isosurfaces of the scalar order parameter with $S = 0.12$ in (b) and $S = 0.2$ in (a) and (c). The surfaces drawn in red represent isosurfaces of $S = 0.3$. Director field in the midplane (parallel to the patterned surfaces) of the simulation box is shown in green. 2×2 unit cells are shown.

Figure 4. (a) Square skyrmion lattice obtained using Pattern S1 with $d = 0.5$, $b = 1.2$, $t = b/36$. (b) Square skyrmion lattices obtained using Pattern S1 with $d = 1.0$, $b = 1.2$, $t = b/36$. (c) Director and disclination structure obtained using Pattern S1 with $d = 1.3$, $b = 1.2$, $t = b/36$. Disclinations are drawn (in yellow) as isosurfaces of the scalar order parameter with $S = 0.2$. Director field in the midplane (parallel to the patterned surfaces) of the simulation box is shown in green. 2×2 unit cells are shown.

Figure 5. Free energy density computed as a function of d and b in the case of Pattern S1 with $t = b/36$. Dotted lines represent ranges of b in which structural transitions occur.

Figure 6. (a) Square half-skyrmion lattices obtained using Pattern S2 with $d = 1.0$, $b = 1.2$, $t = b/36$. (b) Director and disclination structure obtained using Pattern S2 with $d = 1.3$, $b = 1.2$, $t = b/36$. Disclinations are drawn (in yellow) as isosurfaces of the scalar order parameter with $S = 0.2$. Director field in the midplane (parallel to the patterned surfaces) of the simulation box is shown in green. 2×2 unit cells are shown.

Figure 7. Triangular skyrmion lattices obtained using Pattern H1 with $d = 0.3$, $b = 1.4$, $t = b/36$. Disclinations form a honeycomb lattice. Disclinations are drawn (in yellow) as isosurfaces of the scalar order parameter with $S = 0.2$. Director field in the midplane (parallel to the patterned surfaces) of the simulation box is shown in green. 1×2 unit cells are shown.

Figure 8. Disclination network obtained using (a) Pattern H1 and (b) Pattern H2 with $d = 1.0$, $b = 1.4$, $t = b/36$. The honeycomb defect lattice disassembles. Disclinations are drawn (in yellow) as isosurfaces of the scalar order parameter with $S = 0.2$. 1×2 unit cells are shown.

Figure 9. Free energy density as a function of d and b in the case of Pattern H1 with $t = b/36$. Dotted lines represent ranges of b in which structural transitions occur.

Figure 10. Disclination structure obtained using Pattern H2 with $d = 0.3$, $b = 2.0$, $t = b/36$. Hexagonal structure cannot form without the planar strips that exist in Pattern H1. Disclinations are drawn (in yellow) as isosurfaces of the scalar order parameter with $S = 0.2$. Director field in the midplane (parallel to the patterned surfaces) of the simulation box is shown in green. 1×2 unit cells are shown.