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Continuous fatigue damage prediction of a rubber fibre composite structure using multiaxial energy based approach

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Abstract

This paper details an advanced method of continuous fatigue damage prediction of rubber fibre composite structures. A novel multiaxial energy-based approach incorporating a mean stress correction is presented and also used to predict the fatigue life of a commercial vehicle air spring. The variations of elastic strain and complementary energies are joined to form the energy damage parameter. Material parameter α is introduced to adapt for any observed mean stress effect as well as being able to reproduce the well-known Smith-Watson-Topper criterion. Since integration to calculate the energies is simplified, the approach can be employed regardless of the complexity of the thermo-mechanical load history. Several numerical simulations and experimental tests were performed in order to obtain the required stress-strain tensors and the corresponding fatigue lives, respectively. In simulations, the rubber material of the air spring was simulated as nonlinear elastic. The mean stress parameter α , which controls the influence of the mean stress on fatigue life, was adjusted with respect to those energy life curves obtained experimentally. The predicted fatigue life and the location of failure are in very good agreement with experimental observations.

Keywords: Fibre-reinforced composite; Energy approach; Multiaxial fatigue criterion; Mean stress effects; Fatigue life prediction; Finite element analysis

Nomenclature

C

Complementary energy

d	Equivalent cycle damage
$D(C_{ae})$	Fatigue damage due to complementary energy
$D_{ae} = D(U_{ae})$	Fatigue damage
$D_{ae} = D(U_e)$	Fatigue damage
$D(W_{ae})$	Fatigue damage due to strain energy
$D_f = D(U_e)$	Accumulated fatigue damage
N	Cycles to failure
p	Principal component index
R	Stress ratio
U	Total energy
$U_{a,p}$	Total principal energy amplitude
U_{ae}	Total equivalent energy amplitude
$U_{ae,p}$	Total equivalent principal energy amplitude
U_e	Total equivalent energy
$U_{e,p}$	Total equivalent principal energy
$U_{m,p}$	Total mean principal energy
$U_{o,p}$	Origin of total equivalent principal energy
W	Strain energy density
α	Mean stress parameter
$\mathcal{D}$	Damage operator
ε_a	Cycle strain amplitude
ε_{ij}	Strain tensor
ε_p	Principal strain tensor component
σ_a	Cycle stress amplitude
σ_{ij}	Stress tensor
σ_m	Cycle stress mean
σ_{MIN}	Minimum stress experienced during a cycle
σ_{MAX}	Maximum stress experienced during a cycle

1.

2. Introduction

Hyper-elastic materials are used in many forms and chemical mixtures and are found in a broad range of consumer and industrial products such as air springs, tyres, shock absorbers, conveyors and power transmission belts. The versatility of rubber composites is due to their rather special mechanical properties i.e. exceptional elasticity, deformability, damping performance etc. when compared with more traditional metal construction materials.

Fatigue and structural degradation related issues of rubber composites remain an important factor when designing a new product, since normally rubber components are subjected to arbitrary dynamic loadings under different, and often detrimental environmental conditions that cannot be quantified easily. Therefore, the development of numerical approaches that can be used to predict the fatigue life of such parts is a very active area of research in both industry and academia. These approaches¹⁻⁵ are of particular interest in industry where components can be designed and evaluated prior to physical testing⁶. In the long run this results in a more efficient and sustainable product with considerable benefits for end-users in terms of cost, safety and durability.

In recent papers⁷⁻⁹ the fatigue life prediction of filled rubber was treated as the crack nucleation life, obtained from available stress and strain histories. This stress-strain approach is widely used for fatigue life evaluations^{3,10-12}. Here, we assume that functionality is compromised even if a very small crack or hole appears on the surface of the rubber component, resulting in a significant drop in internal pressure, damping properties etc. Since the ratio between the size of the initial crack and the size of the rubber component is usually substantial, the stress-strain approach has proved viable^{8,9}.

The chemical composition of rubber has a direct effect on durability. For example, it is known that the mechanical properties of rubber can be significantly changed during the manufacturing process by adding chemical products such as antioxidants, antiozonants and curatives^{13,14}. Here, the influence of these generally beneficial products on fatigue life is considered indirectly e.g. over the obtained material parameters, measured material response and fatigue curves. The degradation of microstructure due to thermal, environmental (oxygen,

ozone and ultraviolet radiation) and chemical (oil and sour gasoline) factors should not be underestimated¹⁵⁻¹⁷. It turns out that the individual contributions of heat, environmental factors and chemistry on the fatigue life of a rubber component are very difficult to consider. Therefore, environmental effects are not directly considered in the procedure presented in this paper.

When developing the exact method for calculating the fatigue life of an air-spring it is difficult to choose the correct damage parameter. For example, those calculations based on signed von Mises stress have proved⁸ inadequate since the first and the third principal stresses in absolute size can be alike. This can lead to an incorrect calculation of the equivalent stress sign, thus small variations of the stress values can cause a variation of the sign and consequently miscalculation of stress amplitudes and mean stresses. Recently, the critical plane approach was incorporated into a method for calculating the fatigue damage of an air-spring⁸. Evaluation of this method showed^{8,18} that the critical plane approach gives satisfactory results when compared with experimental observations. However, the numerical analysis can be very time consuming if damage is to be calculated for a large number of finite element nodes.

Other approaches exist that can be used to calculate fatigue life¹⁹⁻²¹, one type of which are energy approaches^{3,4,19,22,23}. Overall, energy approaches show good agreement with experiments especially when applied to traditional materials such as steel and steel alloys. Energy approaches are generally based on the assumption that the thermodynamic state of a material can be described in terms of an energy potential that changes when the material undergoes a specific transformation, for example rearrangements of the microstructure due to an external load. If interpreted correctly the difference of energy potential can be used as a parameter to evaluate fatigue damage. However, calculating fatigue life in terms of energy can quickly become very challenging since defining energy potential is not so straightforward. The procedure normally requires the identification of complex analytical mathematical expressions that define thermodynamic variables governing the energy potential of the solid for any given time instant. This often generates further difficulties because the necessary material parameters are difficult to obtain experimentally.

Nevertheless, there are simplified approaches to obtain the energy potential e.g. strain energy density integration approaches³. In order to improve fatigue life predictions, energy approaches also rely on semi-empirical mathematical expressions and dedicated energy parameters derived from experiments²⁴. To this end, several authors have developed energy

approaches that employ the ideas of traditional stress or strain approaches, in particular the influence of the stress amplitude and the mean stress⁴.

In this paper a novel energy based fatigue life approach²⁵ that was primarily developed for application to elastic and hyper-elastic materials is utilised and further enhanced. An overview of the calculation procedure of the energy approach is shown in Figure 1. The procedure can be summarised as follows. Stress and strain tensors and the associated principal stresses and strains are obtained in prior by i.e. finite element analysis (FEA). To do so, any form of the established constitutive model for stress-strain simulation of hyper-elastic materials can be used like polynomial, Marlow or Ogden form. In order for the developed approach to work principal stresses and strains, need to be properly sorted as explained later in this paper. The developed energy damage parameter with the proposed mean stress amendment is calculated for principal stresses and strains in every time instant of the loading history. Mechanical fatigue tests are required to obtain the cyclic damage curve, which are then together with the damage parameter used for prediction of the fatigue life. Details on the developed approach are presented in the following sections. To conclude, the predicted fatigue life is compared against the experimental results obtained for the actual component. It should be noted that the approach is meant to be general, thus multiaxial stress and strain tensors as well as a variable temperature can be applied. In addition, all the variables referred to in this paper are applied to an elastic material; hence “el” superscripts will be omitted for clarity. Unless stated otherwise all variables are time dependent, hence “t” has also been omitted from equations for clarity.

3. Energy damage parameter

The strain energy density W (referred to as strain energy hereafter) is equivalent to the area under the stress-strain curve per unit volume and it is very often used as a measure of fatigue damage^{13,26,27}. The energy can be obtained in post processing e.g. by numerical integration after FEA is carried out and the corresponding stress-strain tensors are available. It should be mentioned, that for variable temperature FEA have to be carried out with temperature dependent material data. The strain energy density can also be determined by analytical expressions and thermodynamic state variables²⁸, but such approaches are generally too complex for practical application. In this paper the strain energy is calculated as follows:

$$W = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij} \quad (1)$$

where for the given interval between the first and a given strain $[0, \varepsilon_{ij}]$ the strain energy W in fact represents the total strain energy. It is assumed, in addition to strain energy, that for every corresponding stress interval $[0, \sigma_{ij}]$, the complementary energy per unit volume (referred to as complementary energy hereafter) can be obtained. Further discussion on such total energy partition is provided elsewhere²⁵, here only the relevant equations for the energy damage parameter calculation are repeated. The complementary energy C is calculated by

$$C = \int_0^{\sigma_{ij}} \varepsilon_{ij} d\sigma_{ij} \quad (2)$$

Figure 2 shows that the sum of W and C formed by the i -th and j -th tensor components always forms a rectangular shape that corresponds to the total energy U .

$$U = W + C \quad (3)$$

This is an important assumption as the total energy includes the contribution of the stress and strain tensor while the necessary integration can be omitted due to simplification with the rectangular shape at any given time (see Figure 2). It should be noted that such approach is feasible although complementary part has no actual physical background and as such do not contribute to total damage^{25,26}.

Both the strain and complementary energies are invariant²⁹ thus stress and strain tensors can be replaced by a principal stress tensor σ_p and principal strain tensor ε_p for $p=1,2,3$. This way multiaxiality and the effect of mean stress can be addressed effectively.

It should be pointed out that it is crucial for the approach that principal stresses and strains are sorted correctly prior running the energy calculation. It is expected that they are sorted with respect to individual direction cosines. For example, it is assumed that direction cosines for the first two time instants have been calculated and that a distinct principal index $p=1,2,3$ has been assigned to each principal direction. This first system of principal axes with non-zero stresses will be referred to as the initial principal system and the corresponding principal axes will be referred to as the initial principal axes. Then, for the second time instant, the principal stress with direction cosines closest to the initial $p=1$ unit basis vector is labelled as σ_1 . The same procedure is employed to assign indexes to the other two principal stresses. The concept of the applied sorting algorithm including the anticipated outcome is shown in Figure 3.

The cone about each principal axis in Figure 3a represents all the limiting angles (a combination of three direction cosines that equally divide 3D space) for which the sorting algorithm provides a single solution i.e. each sorted principal direction lies within an angle of 45° about the corresponding initial principal axis. However, in some multiaxial cases with pronounced shear components or a rapidly changing input history, two principal axes can “fall out” of the cones into the unspecified interspace (see Figure 3). In such cases, a solution can be achieved by taking into account the individual value of the direction cosine. This means that the values of direction cosines are sorted in a descending manner as the highest value with respect to the initial principal axis appointed as the principal index p . The notation of principal strains simply mirrors the notation of principal stresses.

Once the principal stresses and strains are sorted by direction cosines, the calculation procedure can continue²⁵. Next, an energy based damage parameter is defined. Following the Eq. 3 the energy state of each material point under a multiaxial stress-state is given by the total equivalent energy U_e , here representing the energy damage parameter. The parameter is obtained by summing all the principal equivalent energies $U_{e,p}$ whereby:

$$U_e = \sum_{p=1}^3 U_{e,p} . \quad (4)$$

In order to account for the mean stress effects the total equivalent principal energy $U_{e,p}$ in each direction is further split into two segments, namely the amplitude value of the total principal energy $U_{a,p}$ and the mean value of the total principal energy $U_{m,p}$. The mean stress parameter α is proposed so as to control the influence of the mean stress effect as follows:

$$U_{ae,p} = U_{a,p} + \alpha U_{m,p} . \quad (5)$$

Where $U_{ae,p}$ represents the mean stress corrected total equivalent principal energy amplitude obtained for every principal direction. Further discussion and theoretical grounds for implementation of such energy damage parameter were presented elsewhere²⁵. $U_{a,p}$ and $U_{m,p}$ are calculated as $U_{a,p} = \sigma_{a,p} \Delta \varepsilon_p$ and $U_{m,p} = \sigma_{m,p} \Delta \varepsilon_p$, respectively, for $\sigma_{m,p} + \sigma_{a,p} > 0$, otherwise $U_{a,p} = U_{m,p} = U_{ae,p} = 0$. The range $\Delta \varepsilon_p = 2\varepsilon_{a,p}$ is applied to account for the double strain amplitude valid for a fully reversed cycle²⁵.

If $\alpha = 1$, the damage parameter is equivalent to the established Smith-Watson-Topper's parameter³¹ and if $\alpha = 0$, the effect of mean stress is excluded from the calculation; otherwise, if $0 < \alpha < 1$, the effect of mean stress is adjusted with respect to the material response observed in tests at different mean stress values. It should be noted that under a sufficient tensile mean stress load some hyper-elastic materials, like natural rubber and synthetic rubbers similar to the EPDM, can undergo a phase transition where amorphous structure is rearranged into crystalline structure by the so-called strain-induced crystallisation³². This effect may have beneficial impact on the local stress-strain state, thus slowing down the fatigue damage evolution. If material has such specific property, α could take a negative value.

For a loading history with constant load cycles both $U_{a,p}$ and $U_{m,p}$ segments are easily obtained because $\sigma_{a,p}$, $\sigma_{m,p}$ and $\varepsilon_{a,p}$ are conveniently known in advanced. For a variable loading, hysteresis could be very complex and some of the hysteresis loop can require more time to close, therefore principal stress and strain components are usually difficult to determine. In this paper, the components are obtained after both, stress and strain origins³⁰ describing the current state of a material have been worked out. Origins can be continuously calculated for every time instant of the loading history by identifying rainflow and Clormann-Seeger load cycles as shown by Nagode³⁰. $U_{e,p}$ from Eq. 4 can then be written as follows:

$$U_{e,p} = U_{o,p} + \Delta U_{ae,p}, \quad (6)$$

where $\Delta U_{ae,p}$ represents the range of mean stress corrected total equivalent principal energy amplitude given by $\Delta U_{ae,p} = 2U_{ae,p}$, with initial value $U_{ae,p}(t_0) = 0$.

In the next section, the value of the parameter U_e is used to predict the fatigue damage.

4. Fatigue damage modelling

The fatigue damage calculation procedure is presented in this section. Initially, it is assumed that the total equivalent energy amplitude U_{ae} can be used to calculate the associated fatigue damage $D_{ae} = D(U_{ae})$, where in accordance with Eq. 3, D_{ae} can be broken down into two parts:

$$D_{ae} = D(U_{ae}) = D(C_{ae}) + D(W_{ae}). \quad (7)$$

$D(C_{ae})$ represents damage due to the total complementary energy and $D(W_{ae})$ represents the damage due to the total strain energy, where accounting for energy contributions in all three principal directions $U_{ae} = \sum_{p=1}^3 U_{ae,p}$. By assuming only the strain part to be responsible for fatigue damage²⁵, whilst the contribution of the complementary part is set to zero, Eq. 7 could be rewritten as follows:

$$D(U_{ae}) = D(W_{ae}). \quad (8)$$

This is an important assumption as it prepares the necessary grounds for the introduction of the developed energy damage operator approach. With this approach cyclic damage evolution is calculated for every time instant by employing the damage operator $\mathcal{D}$ in conjunction with energy life curves as described in detail previously^{25,33,34}. The operator of Prandtl type is essentially used for modelling of hysteresis phenomenon in magnetism²⁷, however the operator has also been modified for simulations of cyclic metal response and durability simulations^{33,34}. Here, the properties of the Prandtl operator are used to automatically process the damage parameter history²⁵. During the procedure, history of the damage parameter is lead to the damage operator and accumulated fatigue damage D_f is obtained as the variation of the damage operator over time given as:

$$D_f = D(t_i) = \text{Var } \mathcal{D}(t_i). \quad (9)$$

Variation is calculated as follows:

$$\text{Var } \mathcal{D}(t_i) = \sum_{j=1}^{n_i} \left| \mathcal{D}(t_j) - \mathcal{D}(t_{j-1}) \right|, \quad (10)$$

for $\mathcal{D}(t_0) = 0$. Expression in Eq. 10 implies that the value of fatigue damage is monotonically increasing which is equivalent to the linear Palmgren–Miner damage accumulation rule⁸.

As the damage parameter history is processed automatically, U_e should replace U_{ae} as follows from Eqs. 4-6. Hence, Eq. 8 can be rewritten in terms of strain energy as

$$D_f = D(U_e) = D(W_e). \quad (11)$$

Energy life curves are needed in order to connect the value of the damage parameter with the observed experimental fatigue response. Energy life curves are constructed from available $S-N$ or $\varepsilon-N$ fatigue curves for different constant temperatures, where stresses or strains, respectively, are replaced by the cycle equivalent energy by using the following relation

$$U_{ae}^{cycle} = 2 \begin{cases} \sigma_a \varepsilon_a & R_\varepsilon = -1 \\ \sigma_a \varepsilon_a + \alpha \sigma_m \varepsilon_a & \text{otherwise} \end{cases}, \quad (12)$$

for $R_\varepsilon = \frac{\sigma_{MIN}}{\sigma_{MAX}}$. U_{ae}^{cycle} represents the total energy which is equivalent to the total energy of a

loading cycle defined by σ_a and ε_a when performing uniaxial fatigue tests. If the fatigue tests are carried out for loadings other than $R_\varepsilon = -1$, value of the U_{ae}^{cycle} needs to be adjusted for a proper mean stress correction²⁵. Cycle equivalent energy is worked out for all available load levels individually. For practical reasons $U_{ae}^{cycle} - N$ (referred to as $U-N$ hereafter) curves should be transformed into energy $U-d$ curves by replacing the number of cycles to failure N by the equivalent cycle damage $d = 1/N$ ³³. Hence, the d is numerical-wise conveniently bounded between 0 and 1, for no damage and complete failure, respectively.

The damage operator approach can process loading histories with variable temperature. Temperature is in simulation considered indirectly by Prandtl densities which are gained from the available temperature dependent fatigue curves. Further details on proper temperature amendment of the operator as well as the discussion on the assumed application are explained elsewhere²⁵. To this end, in order to use the above presented energy approach to predict the fatigue life of a component, the main steps are summarised again into the following list:

1. Loading history is processed and the corresponding stress and strain tensors are gained from FEA, principal stresses and principal strains are calculated.
2. Temperature dependent stress or strain fatigue curves are transformed into corresponding energy life curves and a proper mean stress parameter is selected (see Eq. 12).
3. Fictive yield energies are calculated²⁵.
4. Principal stresses and principal strains are sorted by direction (see Figure 3).
5. Stress and strain amplitudes and mean stresses are calculated for every principal component and time instant by employing stress, strain origins³⁰.

6. The mean stress corrected total equivalent principal energy (see Eq. 5) and the range of mean stress corrected total equivalent principal energy amplitude is calculated (see Eq. 6).
7. The total equivalent principal energy (see Eq. 4) and the total equivalent energy is calculated (see Eq. 6).
8. Equivalent cyclic damage and the cumulative fatigue damage is calculated.
9. Fatigue life is predicted.

5. Example

In this section, the presented energy damage approach is used to predict the fatigue life of an actual component. The component under investigation is a suspension air spring. The material data assessment and the required numerical simulations are discussed first. Then, fatigue damage modelling and comparison between prediction and observed fatigue life is presented.

Air springs are generally assembled from a piston with a bumper, an upper plate and a rubber composite bellows as shown in Figure 4. Though all components of the air spring are important for its proper functionality, here only the rubber bellows is of interest.

The rubber bellows within the air spring is made of an inner rubber sealing layer, several reinforcement layers of nylon corded rubber material and an outer protective rubber layer. The rubber composite of the bellows was modelled as a layered composite with two layers of nylon fibre material and three layers of rubber material⁸. The details regarding the finite element model are depicted in Appendix A. Hyper-elastic rubber properties were simulated by a multi-linear elastic material model, whilst the cord reinforcement material was simulated by a linear orthotropic material model. The restraints and loads of the finite element (FE) model were applied as follows from the actual air spring loading conditions⁸.

Stress and strain tensors needed for subsequent calculation of the total equivalent energy U_e were obtained from finite element analysis (FEA) in the Ansys⁸. It should be noted that due to specific physical properties of rubber and rubberlike materials this kind of materials are usually modelled using hyper-elastic material models. Here, multi-linear elastic material model was used to simulate stress-strain response of rubber part e.g. the air spring bellows. The reason is that in the presented case the elongation of the rubber in the air spring bellows is only up to 80 percent, which is relatively low for this kind of rubber material, hence hyperelasticity is not yet fully expressed. In addition, only uniaxial results of the rubber mixture were available for evaluation. Under these circumstances, it has been shown⁸ that by using a multilinear elastic material model the simulated behaviour (stress-strain response) is

in good agreement with observations. On the contrary, if a hyper-elastic material model is being used and only uniaxial material data is available, simulation can produce unsatisfying results. The main reason is that uniaxial material data is generally not enough to accurately determine parameters of most of hyper-elastic models with exception of the Marlow model that was at the time of the FEA not available in the Ansys. The applied material data including multilinear response curve of rubber material and orthotropic material properties of cord material are presented in Appendix B.

Stress and strain tensors were exported for the inner layer as the corresponding calculated stress values are considerably higher as compared to all other layers which is why the critical damage was also observed on the inner layer. Principal values were sorted by direction as required for the energy calculation. Loading conditions with measured dynamic pressure characteristic of the air spring is shown in Figure 5a. In addition Figure 5b shows an example of comparison between direction based and value based sorted principal stresses on actual node of the bellows.

In order to gain the energy life curve, air springs were physically tested by seven preselected displacement amplitudes at different design heights and internal pressure⁸ until the point of crack initiation. For each load case four specimens were tested by using dedicated testing devices. It should be noted that for the purposes of this paper only the observed fatigue lives are provided (Table 1), whilst experimental procedure and the equipment used are presented elsewhere⁸. To assemble the energy life curve shown in Figure 7, only values on the critical node, that is the node with the maximum total equivalent energy amplitude, are used. The procedure is similar to the construction of Wöhler curves with at least two load levels (see Figure 6).

To properly account for the mean stress effect, the optimal value of α from Eq. 5 was searched for. It is assumed that the optimal value of α is attained when a maximum R^2 ³⁵ of the test data is reached as depicted in Figure 7. It turns out that the optimal value here is $\alpha = 1$ for a maximum $R^2 = 0.8783$.

Fatigue damage and fatigue life can be automatically calculated by considering the proper energy damage curve only after the total strain energy history is obtained. This means that material parameters and material model related data need to be worked out prior to the damage calculation. For validation purposes, four air springs were tested using linear amplitude load spectra as presented in Table 2. Table 2 shows the number of finished cycle blocks for four air springs at eight different load levels. Loads were applied sequentially. All four air springs were exposed to the same number of cycle blocks up to an amplitude of 50

mm. The remaining life of each air spring was tested with an amplitude of 55 mm. Values for an amplitude of 55^a mm indicates completed cycle blocks until the start of the leakage (this state reflects the appearance of initial fatigue crack with no significant impact on air spring function). Values for an amplitude of 55^b mm indicate an additional cycle block until the critical damage – measured by a 2 bar drop of internal pressure. At this point, when the air spring loses its function, the test was stopped and fatigue damage was assumed to equal 1. It should be noted that considerable scatter of a number of cycle blocks occurs at 55^a and 55^b mm, due mainly to the air spring material and geometric inconsistencies caused by the manufacturing process.

6. Discussion

In order to evaluate the fatigue damage prediction quantitatively, a comparison between the calculated energy based fatigue damage presented here and a previously presented critical plane based predicted fatigue damage was made⁵. Results are presented by the contour plots as shown in Figure 8. The critical plane calculation method requires significantly more computational effort as compared to the energy calculation method, since damage is calculated for every of the 128th plane in each node; thus only a single column of elements including the critical one with maximum stresses was considered for fatigue calculation. Consequently, Figure 8a shows a solution for the said single column of elements with results copied to all other elements of the air spring model. For the energy approach, a solution was obtained for all nodes as shown in Figure 8b. Furthermore, the fatigue damage distribution varies from side to side of the air spring bellows revealing the effect of the applied boundary conditions (Figure 7). The maximum value was obtained at the same location by both analyses.

Numerical predictions were validated against the experimentally observed fatigue damages. The predicted and the observed fatigue damage values at the end of the loading sequence (complete loading spectra as schematically depicted in Figure 5 was considered) when the air spring failed are collected in Table 3.

In general, the closer the predicted fatigue damage is to the value of 1.00, the more accurate the prediction is. In that sense, Table 3 shows that simulations using the energy approach result in a better prediction (less conservative) as compared to predictions made by the critical plane approach - mostly due to a different consideration of stress and strain multiaxiality. Specifically, no transformation of stress and strain tensors into their uniaxial equivalents is required by the energy approach.

7. Conclusions

An advanced energy-based fatigue damage operator approach including the mean stress effect was presented and verified in this paper. The predicted fatigue life shows good agreement with tested data. In fact, using the energy approach results in a less conservative prediction as compared to prediction made by the critical plane approach, 1.11 and 1.46 respectively.

The main advantages of the energy-based approach are:

- Strain energy including the mean stress effect is used as a fatigue damage parameter.
- Stress and strain tensors are transformed into principal values, sorted and used in the calculation procedure directly. No equivalent value or critical plane approach is required. The proposed sorting is based on the algorithm that continuously follows the principal directions and properly prescribes the principal notations.
- The energy calculation is simplified by using energy partition e.g. strain and complementary energy.
- Strain energy is obtained for every time instant regardless of the complexity and length of the loading history.
- The effect of mean stress is taken into account automatically by introducing a material parameter α . If $\alpha = 0$ mean stress effect is not considered and if $\alpha = 1$ the damage parameter results in the established Smith-Watson-Topper parameter. The approach can also take α that is more than 1 or less than 0, if experimental observations reveal such mean stress dependencies.
- The energy approach used here requires less computational effort as compared to the critical plane approach as energy is calculated in every node for every time instant only once.
- This approach offers online fatigue damage calculation, meaning that damage is obtained continuously regardless of the complexity and length of loading history.
- Assuming the FEA for the product under consideration is made correctly (gained tensors reflect actual stress-strain state), the approach is applicable for rubbers as well as for metallic alloys.

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9. References

1. Andriyana A, Verron E. Prediction of fatigue life improvement in natural rubber using configurational stress. *Int J Solids Struct* 2007; 44 (7-8): 2079-2092.

2. Woo CS, Kim WD, Lee SH, C BI, Park HS. Fatigue life prediction of vulcanized natural rubber subjected to heat-aging. *Procedia Eng* 2009; 1: 9-12.
3. Mars WV, Fatemi A. A literature survey on fatigue analysis approaches for rubber. *Int J Fatigue* 2002; 24: 949-961.
4. Park J, Nelson D. Evaluation of an energy-based approach and a critical plane approach for predicting constant amplitude multiaxial fatigue life. *Int J Fatigue* 2000; 22: 23-39.
5. Pidaparti RMV, May AW. Micromechanical analysis of fatigue cracks in cord-rubber composites. *Compos Struct* 2001; 54: 459-465.
6. Li F, Liu J, Lu Y, Zhang L. Numerical simulation and experimental verification of heat build-up for rubber compounds. *Polymer* 2016; 101: 199-207.
7. Oman S, Nagode M, Fajdiga M. The material characterization of the air spring bellow sealing layer. *Mater. Design* 2009; 30: 1141-1150.
8. Oman S, Fajdiga M, Nagode M. Estimation of air-spring life based on accelerated experiments. *Mater Design* 2010; 31: 3859-3868.
9. Oman S, Nagode M. On the influence of the cord angle on air-spring fatigue life. *J Eng Fail Anal* 2013; 27: 61-73.
10. Williams JG. *Fracture mechanics of polymers*. Chichester: Ellis Horwood, 1984.
11. Hardy D, Moneypenny H, Holderied M, Harris J. Effect of waveform on crack growth. *Rubber Chem* 1999; 99.
12. South JT, Scott W, Reifsnider KL, Reifsnider CL. Crack growth of natural rubber using a modified double cantilever beam. *Mech Mater* 2002; 34(8): 451-458.
13. Mars WV, Fatemi A. Fatigue crack nucleation and growth in filled natural rubber. *Fatigue Fract Engng Mater Struct* 2003; 26: 779-789.
14. Mars WV, Fatemi A. Factors that affect the fatigue life of rubber: a literature survey. *Rubber Chem Technol* 2004; 77(3): 391-412.
15. Lake GJ, Thomas AG, Roberts AD. *Natural rubber science and technology*. Oxford: Oxford University Press, 1988.
16. Lake GJ, Lindley PB. The mechanical fatigue limit for rubber. *J. Appl Polym Sci* 1965; 9(4): 1233-1251.
17. Gent AN. Some chemical effects in fatigue cracking of vulcanized rubbers. *J Appl Polym Sci* 1962; 6(23): 497-502.
18. Fatemi A, Socie DF. A critical plane approach to multiaxial fatigue damage including out-of-phase loading. *Fatigue Fract Engng Mater Struct* 1988; 11: 149-165.

19. Zine A, Benseddiq N, Nait-Abdelaziz M. Rubber fatigue life under multiaxial loading: Numerical and experimental investigations. *Int J Fatigue* 2011; 33(10): 1360-1368.
20. Grandcoin J, Boukamel A, Lejeunes S. A micro-mechanically based continuum damage model for fatigue life prediction of filled rubbers. *Int J Solids Struct* 2014; 51(6): 1274-1286.
21. Ayoub G, Nait-Abdelaziz M, Zairi F, Gloaguen JM, Charrier P. Fatigue life prediction of rubber-like materials under multiaxial loading using a continuum damage mechanics approach: effects of two-blocks loading and R ratio. *Mech Mater* 2012; 52: 87-102.
22. Li F, Liu J, Mars WV, Chan TW, Lu Y, Yang H, Zhang L. Crack precursor size for natural rubber inferred from relaxing and non-relaxing fatigue experiments. *Int J Fatigue* 2015; 80: 50-57.
23. Shangguan WB, Wang XL, Deng JX, Rakheja S, Pan XY, Yu B. Experiment and modeling of uniaxial tension fatigue performances for filled natural rubbers. *Mater Design* 2014; 58: 65-73.
24. Wang B, Lu H, Kim GH. A damage model for the fatigue life of elastomeric materials. *Mech Mater* 2002; 34(8): 475-483.
25. Nagode M, Šeruga D. Fatigue life prediction using multiaxial energy calculations with the mean stress effect to predict failure of linear and nonlinear elastic solids. *Results Phys* 2016; 6: 352-364.
26. Charkaluk E, Constantinescu A. Dissipative aspects in high cycle fatigue. *Mech Mater* 2009; 41 (5): 483-494.
27. Visintin A. Differential models of hysteresis. In: Antman SS et al, editor. *Applied Mathematical Sciences Vol. 111*, Berlin: Springer, 1994.
28. Lemaitre J., Chaboche J. L. *Mechanics of solid materials*. Cambridge: Cambridge university press, 2002.
29. Ottosen NS, Ristinmaa M. *The mechanics of constitutive modelling*. Oxford: Elsevier, 2005.
30. Nagode M. Continuous damage parameter calculation under thermo-mechanical random loading. *MethodsX* 2014; 1: 81-89.
31. Smith KN, Watson P, Topper TH. A stress–strain function for the fatigue of metals. *J Mater* 1970; 5(4): 767-778.
32. Abraham F., Alshuth T., Jerrams S. Dependence on mean stress and stress amplitude of fatigue life of EPDM elastomers. *Plast Rubber Compos* 2001; 30 (9): 421-425.

33. Nagode M, Hack M, Fajdiga M. High cycle thermo-mechanical fatigue: Damage operator approach. *Fatigue Fract Engng Mater Struct* 2009; 32 (6): 505-514.
34. Nagode M, Hack M, Fajdiga M. Low cycle thermo-mechanical fatigue: damage operator approach. *Fatigue Fract Engng Mater Struct* 2009; 33 (3): 149-160.
35. Rice JA. *Mathematical Statistic and Data Analysis* 3rd Ed. Belmont: Brooks-Cole Cengage Learning, 2007.

Appendix A: Details of the finite element model

Finite element model - mesh

Finite element model - contact pairs

^I Referring to the Ansys internal notation of the four-node element with six degrees of freedom at each node.

^{II} Referring to the Ansys internal notation of the layered version of the eight-node structural solid with three degrees of freedom at each node.

^{III} Referring to the Ansys internal notation of the linear two-node beam element in 3-D with six degrees of freedom at each node.

^{IV} Referring to the Ansys internal notation of the surface-to-surface contact elements.

Appendix B: Rubber and cord material parameters

Captions for Figures and Tables

Figure 1: Overview of the calculation procedure using the multiaxial energy approach for fatigue life prediction

Figure 2: Example of the graphical interpretation of total energy U for the known (FEA) hyper-elastic stress-strain material response.

Figure 3: Limiting cones about principal axes (a) and comparison between value and direction cosines sorted principal stresses (b)

Figure 4: The finite element model of the rubber air spring bellows

Figure 5: Loading conditions with measured dynamic pressure characteristic (a) and an example of comparison between direction based and value based sorted principal stresses on actual node

Figure 6: Overview of the procedure to obtain stress and strain histories

Figure 7: Energy life curve (a) and the corresponding energy damage curve (b)

Figure 8: Results of fatigue damage calculation for the assumed loading spectra and average cycle blocks to failure obtained by (a) the energy approach and (b) the critical plane approach⁸ (symmetrical view cut)

Table 1: The fatigue test results in number of cycle blocks to failure for constant displacement amplitude

Table 2: The fatigue test results in number of cycle blocks to failure for loading history with variable amplitude

Table 3: Comparison between calculated fatigue damage

FE model (symmetrical view cut, fully loaded state)

Test Number		ZV-0-001	ZV-0-002	ZV-0-002	ZV-0-003	ZV-0-004	ZV-0-005	ZV-0-006
Design height (mm)		205.5	214.4	214.4	214.4	205.5	214.4	214.4
Load amplitude (mm)		44.5	35.6	35.6	50.7	31.5	50.7	44.5
Pressure at DH (bar)		9.0	9.0	9.0	9.0	9.0	7.0	9.0
Number of cycle blocks to failure (2 bar pressure drop)	Specimen 1	1293	1530	1564	608	494	3116	1157
	Specimen 2	942	733	1203	870	2297	4342	655
	Specimen 3	1568	1553	573	1008	1769	2688	1741
	Specimen 4	1527	918	1887	1052	1312	3447	1508
	Average	1333	1184	1307	885	1468	3394	1265
	St. dev.	287	420	563	200	764	694	472
Number of cycle blocks to failure (leakage start)	Specimen 1	377	713	743	250	344	1599	597
	Specimen 2	377	708	389	400	1696	4104	320
	Specimen 3	1042	975	308	693	669	1073	1393
	Specimen 4	1069	574	1218	693	575	2545	1336
	Average	716	743	665	509	821	2330	912
	St. dev.	392	168	415	221	599	1330	536
Inner layer crack position (mm) (measured from the top of the piston)	Specimen 1	–	43	–	–	–	46	44
	Specimen 2	–	48	–	–	–	42	48
	Specimen 3	–	49	–	–	–	50	45
	Specimen 4	–	40	–	–	–	46	47
	Average	–	45	–	–	–	46	46
	St. dev.	–	4.2	–	–	–	3.2	1.8

Amplitude (mm)	Specimen				Average	St. dev.
	1	2	3	4		
	<i>Number of finished cycle blocks</i>					
20	639.6	639.6	639.6	639.6	639.6	0
25	391.2	391.2	391.2	391.2	391.2	0
30	239.3	239.3	239.3	239.3	239.3	0
35	146.4	146.4	146.4	146.4	146.4	0
40	89.5	89.5	89.5	89.5	89.5	0
45	55.3	55.3	55.3	55.3	55.3	0
50	34.1	34.1	34.1	34.1	34.1	0
55 ^a	78.5	228.6	182.3	592.3	270.4	223.6
55 ^b	246	353.8	468.8	709.9	444.6	198.9

^a Number of cycle blocks to leakage start.

^b Number of cycle blocks to 2 bar pressure drop.

<i>Approach</i>	Energy	Critical plane	Observed
Predict. fat. damage [/]	1.11	1.46	1.00